JEE (ADVANCED) 2023 PAPER-2

[PAPER WITH SOLUTION]

HELD ON SUNDAY 04TH JUNE 2023

PHYSICS

SECTION 1 (Maximum Marks : 12)

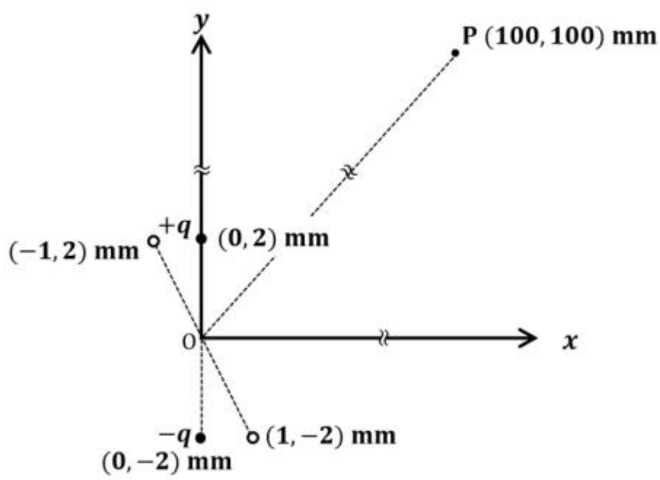
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If **ONLY** the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases.

[:Q.1] An electric dipole is formed by two charges $+q$ and $-q$ located in xy-plane at (0,2) mm and (0, −2) mm, respectively, as shown in the figure. The electric potential at point P (100,100) mm due to the dipole is V_0. The charges $+q$ and $-q$ are then moved to the points (−1, 2) mm and (1, −2) mm, respectively. What is the value of electric potential at P due to the new dipole?



[:A] $V_0/4$

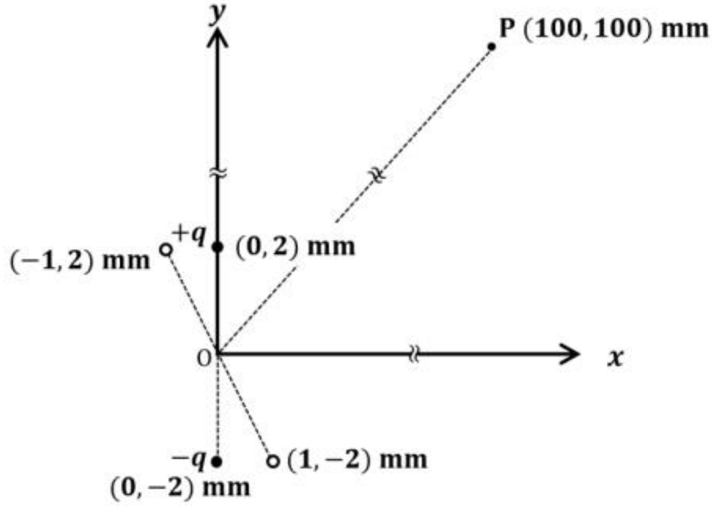
[:B] $V_0/2$

[:C] $V_0/\sqrt{2}$

[:D] $3V_0/4$

[:ANS] B

[:SOLN]



$$V = \frac{k(\vec{p}\cdot\vec{r})}{r^3}$$

$$\vec{p} = q \times (4 \times 10^{-3})\hat{j}$$

$$\vec{r} = 0.1\hat{\ell} + 0.1\hat{j}$$

$$V_0 = \frac{k\,a(4\times10^{-3})\hat{j}\cdot(0.1\hat{\ell}+0.1\hat{j})}{(\sqrt{2}\times0.1)^3}$$

$$= \frac{k\,Q\times4\times10^{-4}}{2\sqrt{2}\times10^{-3}} \quad \text{....... (i)}$$

Now, After rotation

$$\vec{P}- = q\left[-2\times10^{-3}\hat{\ell} + 4\times10^{-3}\hat{j}\right]$$

$$\vec{r} = 0.1\hat{\ell} + 0.1\hat{j}$$

$$V = \frac{k(\vec{p}\cdot\vec{r})}{r^3} = \frac{kq\left[-2\times10^{-4}+4\times10^{-4}\right]}{2\sqrt{2}\times10^{-3}}$$

$V = \dfrac{kq \times 2 \times 10^{-4}}{2\sqrt{2} \times 10^{-3}}$ ...... (ii)

From equation (i) and (ii) ,

$\therefore \quad V = \dfrac{V_0}{2}$

[:Q.2] Young's modulus of elasticity Y is expressed in terms of three derived quantities, namely, the gravitational constant G, Planck's constant h and the speed of light c, as $Y = c^{\alpha}h^{\beta}G^{\gamma}$. Which of the following is the correct option?

[:A] $\alpha = 7, \beta = -1, \gamma = -2$

[:B] $\alpha = -7, \beta = -1, \gamma = -2$

[:C] $\alpha = 7, \beta = -1, \gamma = 2$

[:D] $\alpha = -7, \beta = 1, \gamma = -2$

[:ANS] **A**

[:SOLN] $[\gamma] = ML^{-1}T^{-2}$

$[G] = M^{-1}L^{3}T^{-2}$

$[h] = ML^{2}T^{-1}$

$[c] = LT^{-1}$

$\gamma = c^{\alpha}h^{\beta}G^{\gamma}$

$ML^{-1}T^{-2} = M^{\beta-\gamma}L^{\alpha+2\beta+3\gamma}T^{-\alpha-\beta-2\gamma}$

$\Rightarrow \quad \beta - \gamma = 1$ (i)

$\alpha + 2\beta + 3\gamma = -1$ (ii)

$-\alpha - \beta - 2\gamma = -2$

$\Rightarrow \quad \alpha + \beta + 2\gamma = 2$ (iii)

From equation (i), (ii) and (iii) ,

$\Rightarrow \quad \boxed{\alpha = 7}$

$\boxed{\beta = -1}$

$\boxed{\gamma = -2}$

So, Option (A) is correct.

[:Q.3] A particle of mass m is moving in the xy-plane such that its velocity at a point (x, y) is given as $\vec{v} - = \alpha(y\hat{x} + 2x\hat{y})$, where α is a non-zero constant. What is the force $\vec{F}$ acting on the particle?

[:A] $\vec{F} = 2m\alpha^2(x\hat{x} + y\hat{y})$

[:B] $\vec{F} = m\alpha^2(y\hat{x} + 2x\hat{y})$

[:C] $\vec{F} = 2m\alpha^2(y\hat{x} + x\hat{y})$

[:D] $\vec{F} = m\alpha^2(x\hat{x} + 2y\hat{y})$

[:ANS] **A**

[:SOLN] $\vec{v} - = \alpha(y\hat{x} + 2x\hat{y})$

$v_x = \alpha y$

$v_y = 2\alpha x$

$$\vec{a} = \frac{d\vec{v}}{dt} = \alpha\left[\frac{dy}{dt}\hat{x} + 2\frac{dx}{dt}\hat{y}\right]$$

$$\vec{a} = \alpha\left[(2\alpha x)\hat{x} + 2 \times \alpha y\hat{y}\right]$$

$$\vec{a} = 2\alpha^2\left[x\hat{x} + y\hat{y}\right]$$

$$\vec{F} = m\vec{a} = 2m\alpha^2\left[x\hat{x} + y\hat{y}\right]$$

[:Q.4] An ideal gas is in thermodynamic equilibrium. The number of degrees of freedom of a molecule of the gas is n. The internal energy of one mole of the gas is U_n and the speed of sound in the gas is v_n. At a fixed temperature and pressure, which of the following is the correct option?

[:A] $v_3 < v_6$ and $U_3 > U_6$

[:B] $v_5 > v_3$ and $U_3 > U_5$

[:C] $v_5 > v_7$ and $U_5 < U_7$

[:D] $v_6 < v_7$ and $U_6 < U_7$

[:ANS] **C**

[:SOLN] For n degree of freedom and n' no of mole

$\Rightarrow$ Internal energy $U = \frac{n}{2} \cdot n'RT$

$\Rightarrow$ Velocity of sound

$$V=\sqrt{\frac{\gamma RT}{M_0}},\ \gamma=1+\frac{2}{n}$$

$\Rightarrow$ For $n \rightarrow$ more, $\gamma \rightarrow$ less, $V_{sound} \rightarrow$ less, $U \rightarrow$ more

$\therefore$ (C)

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

  Zero Marks : 0 If unanswered;

  Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks; choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

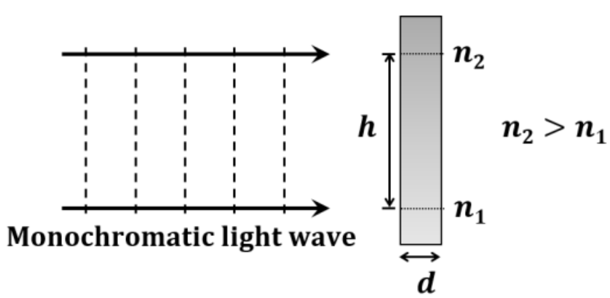
  choosing ONLY (B) will get +1 mark;

  choosing ONLY (D) will get +1 mark;

  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

  choosing any other option(s) will get −2 marks.

[:Q.5] A monochromatic light wave is incident normally on a glass slab of thickness d, as shown in the figure. The refractive index of the slab increases linearly from n_1 to n_2 over the height h. Which of the following statement(s) is(are) true about the light wave emerging out of the slab?



[:A] It will deflect up by an angle $\tan^{-1}\left[\frac{(n_2^2 - n_1^2)d}{2h}\right]$.

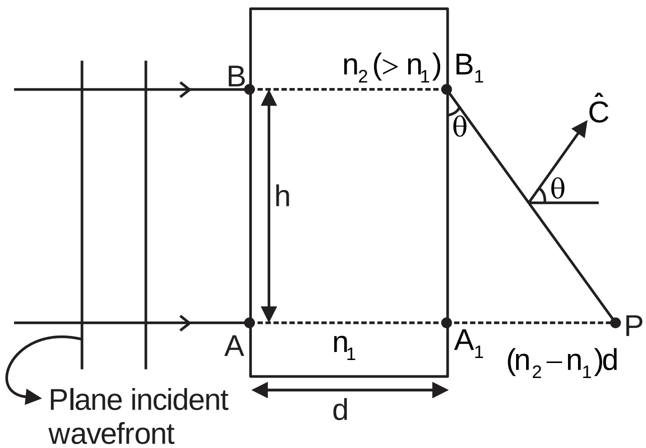
[:B] It will deflect up by an angle $\tan^{-1}\left[\frac{(n_2 - n_1)d}{h}\right]$.

[:C] It will not deflect.

[:D] The deflection angle depends only on $(n_2 - n_1)$ and not on the individual values of n_1 and n_2.

[:ANS] B, D

[:SOLN]



⇒ Time taken to wave front at point B reach at end of slab

$$t_2 = \frac{d}{v = \frac{c}{n_2}} = \frac{n_2 d}{c}$$

⇒ Time taken to wave front at point A reach at end of slab

$$t_2 = \frac{d}{v = \frac{c}{n_1}} = \frac{n_1 d}{c}$$

⇒ As $n_2 > n_1$, $t_2 > t_1$

⇒ Hence when wavefront at point B reaches at end of slab the wavefront at A reaches the point P.

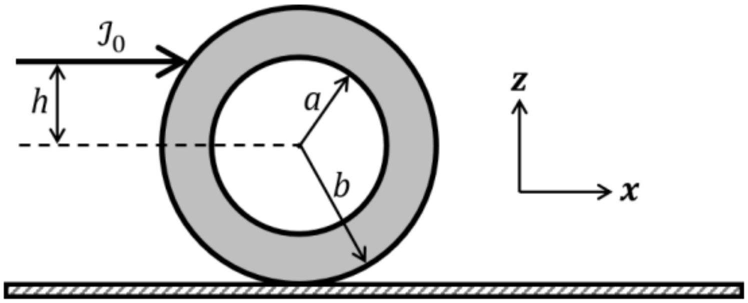
⇒ With $A_1P = C(t_2 - t_1) = C \cdot \left[\frac{n_2 d}{c} - \frac{n_1 d}{c}\right] = (n_2 - n_1)d$

⇒ Angle of deflection

$$\tan\theta = \frac{A_1P}{A_1B_1} = \frac{(n_2 - n_1)d}{h}$$

Ans. (B), (D)

[:Q.6] An annular disk of mass M, inner radius a and outer radius b is placed on a horizontal surface with coefficient of friction μ, as shown in the figure. At some time, an impulse $J_0\hat{x}$ is applied at a height h above the center of the disk. If $h = h_m$ then the disk rolls without slipping along the x-axis. Which of the following statement(s) is(are) correct?



[:A] For $\mu \neq 0$ and $a \to 0$, $h_m = b/2$.

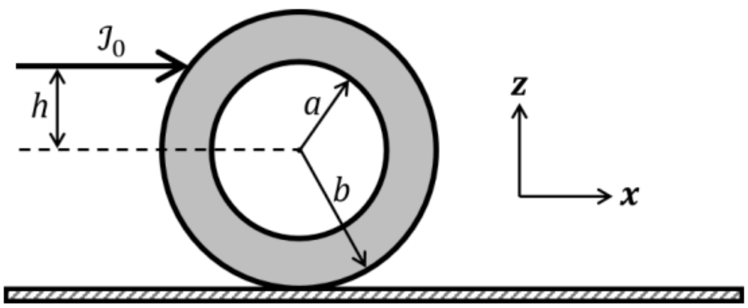
[:B] For $\mu \neq 0$ and $a \to b$, $h_m = b$.

[:C] For $h = h_m$, the initial angular velocity does not depend on the inner radius a.

[:D] For $\mu = 0$ and $h = 0$, the wheel always slides without rolling.

[:ANS] **A, B, C, D**

[:SOLN]



$\Rightarrow$ For $h = h_m$ and disc rolls without slipping,

* From linear impulse equation,

$$J_0 = \Delta P$$

$$J_0 = MV \Rightarrow V = \frac{J_0}{M}$$

* From angular impulse equation about cm,

$$\vec{J} = \Delta\vec{L}$$

$$J_0 \times h = I_{cm} \cdot \omega \Rightarrow \omega = \frac{J_0 \cdot h}{I_{cm}}$$

* For pure rolling ,

$V = \omega(r = b)$

$\boxed{\omega = \frac{v}{b} = \frac{J_0}{Mb}}$, Independent of a. Ans. (C)

⇒ For $a \to 0$, Annular disc behaves as disc of radius b

$$\omega = \frac{J_0}{Mb} = \frac{J_0 h}{I_{cm} = \frac{Mb^2}{2}}$$

$\boxed{h = \frac{b}{2}}$ Ans. (A)

⇒ For $a \to b$, Annular disc behaves as ring of radius b

$$\omega = \frac{J_0}{Mb} = \frac{J_0 h}{I_{cm} = Mb^2}$$

$\boxed{h = b}$ Ans. (B)

⇒ For $\mu \to 0, h \to 0$

Wheel always slides without rolling as these is no torque on it.

Ans. (D)

[:Q.7] The electric field associated with an electromagnetic wave propagating in a dielectric medium is given by $\vec{E} = 30(2\hat{x} + \hat{y})\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]\text{Vm}^{-1}$. Which of the following option(s) is(are) correct?

[Given: The speed of light in vacuum, $c = 3 \times 10^8$ m s^{-1}]

[:A] $B_x = -2\times10^{-7}\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]\text{Wbm}^{-2}$

[:B] $B_y = 2\times10^{-7}\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]\text{Wbm}^{-2}$

[:C] The wave is polarized in the xy-plane with polarization angle 30° with respect to the x-axis.

[:D] The refractive index of the medium is 2.

[:ANS] **A, D**

[:SOLN] Given, Electric field

$$\vec{E} = 30(2\hat{x} + \hat{y})\sin\left[2\pi\left(5\times10^{14}t - \frac{10^7}{3}z\right)\right]Vm^{-1}$$

Represent light wave propagating in z-direction with $\vec{E}$-oscillate in x-y plane. i.e. $\vec{E}$ of wave is polarized in x-y plane with polarization angle with x-axis.

$$\tan\theta = \frac{E_y}{E_x} = \frac{1}{2}$$

$$\theta = \tan^{-1}\frac{1}{2}$$

⇒ Refractive index of medium

$$n = \frac{c}{v} = \frac{c}{\left(\frac{\omega}{k}\right)} = \frac{3\times10^8}{\frac{2\pi\times5\times10^{14}}{2\pi\times\frac{10^7}{3}}} = \frac{10}{5} = 2$$

⇒ For light wave,

$$C = \frac{E}{B},\ B_x = \frac{E_x}{C},\ B_y = \frac{E_y}{C}$$

$$B_x = \frac{E_x}{C} = \frac{30\times2}{3\times10^8} = 2\times10^{-7}$$

$$B_y = \frac{E_y}{C} = \frac{30}{3\times10^8} = 1\times10^{-7}$$

Ans. (A)

SECTION 3 (Maximum Marks : 24)

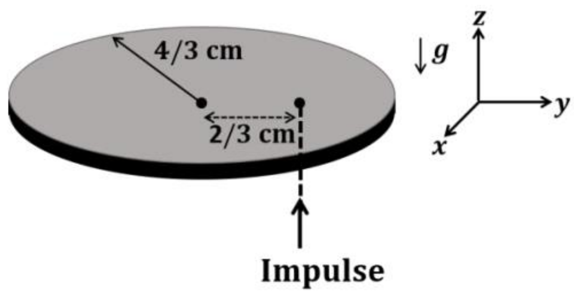
- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If **ONLY** the correct integer is entered;

  Zero Marks : 0 In all other cases.

[:Q.8] A thin circular coin of mass 5 gm and radius 4/3 cm is initially in a horizontal xy-plane. The coin is tossed vertically up ($+z$ direction) by applying an impulse of $\sqrt{\frac{\pi}{2}} \times 10^{-2}$ N-s at a distance 2/3 cm from its center. The coin spins about its diameter and moves along the $+z$ direction. By the time the coin reaches back to its initial position, it completes n rotations. The value of n is ____.

[Given: The acceleration due to gravity g = 10 m s^{-2}]



[:ANS] **30**

[:SOLN] ⇒ From linear impulse equation,

$$J = \Delta \vec{P} = mV$$

$$V = \frac{J}{m} = \frac{\sqrt{\frac{\pi}{2}} \times 10^{-2}}{5 \times 10^{-3}} = \sqrt{2\pi}$$

Time of flight $t = \frac{2V}{g} = \frac{2\sqrt{2\pi}}{g}$

⇒ From angular impulse equation,

$$J = \Delta L$$

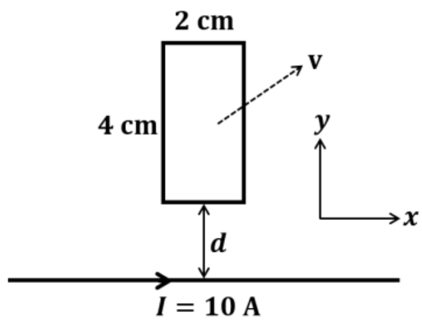
$$J_0 \times \left(\frac{2}{3} \times 10^{-2}\right) = I\omega$$

$$\omega = \frac{\sqrt{\frac{\pi}{2}} \times 10^{-2} \times \frac{2}{3} \times 10^{-2}}{I = \frac{MR^2}{4} = \frac{1}{4} \times 5 \times 10^{-3} \times \left(\frac{4}{3} \times 10^{-2}\right)^2} = \sqrt{\frac{\pi}{2}} \times 3 \times 10^2$$

⇒ No. of rotation $n = \frac{t}{T = \frac{2\pi}{\omega}} = \frac{\frac{2\sqrt{2\pi}}{g}}{\frac{2\pi}{\sqrt{\frac{\pi}{2}} \times 3 \times 10^2}} = 30$

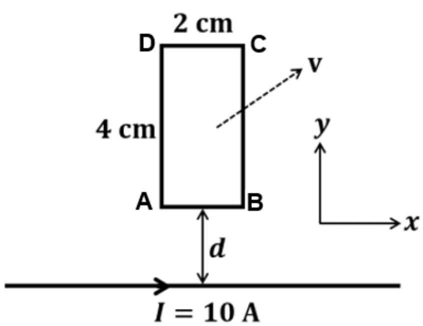
[:Q.9] A rectangular conducting loop of length 4 cm and width 2 cm is in the xy-plane, as shown in the figure. It is being moved away from a thin and long conducting wire along the direction $\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y}$ with a constant speed v. The wire is carrying a steady current I = 10 A in the positive x-direction. A current of 10 μA flows through the loop when it is at a distance d = 4 cm from the wire. If the resistance of the loop is 0.1 Ω, then the value of v is _______ m s^{-1}.

[Given: The permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2}]



[:ANS] **4**

[:SOLN]



Given

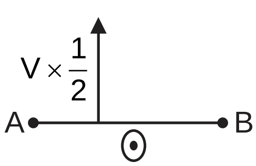
Direction : $\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y}$

Current = $10\mu A$ flows through loop at d = 4

Resistance of loop = $0.1\,\Omega$

Value of V = ?

Induced emf in $AB = (\vec{V} \times \vec{B}) \cdot \ell$



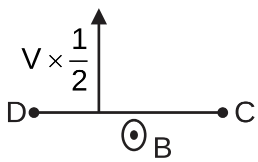
$$B=\frac{\mu_0 \ell^{\circ}}{2\pi r}=\frac{4\pi\times10^{-7}\times10}{2\pi\times4\times10^{-2}}=0.5\times10^{-4}\text{ Tesla}$$

$$\text{Emf in } AB = B\times\frac{1}{2}\times2\times10^{-2}\times V$$

$$=\frac{1}{2}\times10^{-4}\times\frac{1}{2}\times2\times10^{-2}\times V$$

$$=\frac{V}{2}\times10.6\text{ Volt}$$

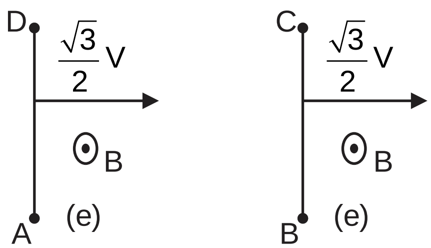
$$\text{Emf in CD, } e_2 = B\times\frac{1}{2}\times2\times10^{-2}\times V$$



$$=\frac{\mu_0\ell'}{2\pi(8\times10^{-2})}\times\frac{1}{2}\times2\times10^{-2}\times V$$

$$=\frac{V}{4}\times10^{-6}\text{ Tesla}$$

emf in BC and AD ,



Both emf will be equal

$\therefore$ emf in loop = $e_1-e_2+e-e \;=e_1-e_2$

$$e_1-e_2=V\times\frac{1}{2}\times10^{-6}-\frac{1}{4}\times10^{-6}\times V\;=\frac{V}{4}\times10^{-6}$$

Resis = $0.1\,\Omega$

$$\text{Current }=\frac{V}{R}=\frac{V\times10^{-6}}{4\times0.1}=\frac{10V}{4}\mu A$$

and $i=10\mu A$

$$\therefore\quad 10=\frac{10}{4}V$$

$$\boxed{V=4m/s}$$

[:Q.10] A string of length 1 m and mass 2×10^{-5} kg is under tension T. When the string vibrates, two successive harmonics are found to occur at frequencies 750 Hz and 1000 Hz. The value of tension T is _____ Newton.

[:ANS] **5**

[:SOLN] Given

$\ell = 1\text{m}$

$\text{m} = 2 \times 10^{-5}\text{ kg}$

Successive harmonic 750 Hz, 1000 Hz

$$\frac{V}{2\ell} = 1000 - 750$$

$$\frac{V}{2\ell} = 250$$

$$V = 2 \times 1 \times 250$$

$$V = 500\text{ m/s}$$

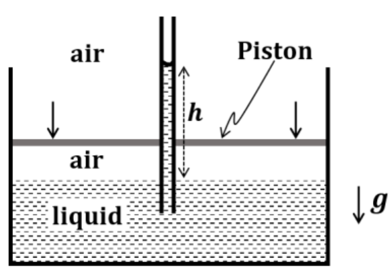
$$\sqrt{\frac{T}{m}} = 500$$

$$T = (500)^2 \times 2 \times 10^{-5}$$

$$T = 500 \times 500 \times 2 \times 10^{-5} \quad = 5\text{N Ans.}$$

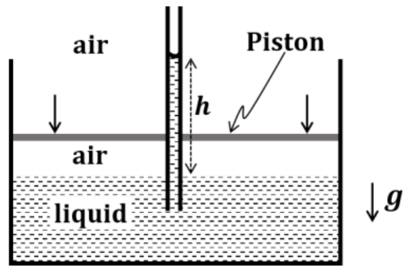
[:Q.11] An incompressible liquid is kept in a container having a weightless piston with a hole. A capillary tube of inner radius 0.1 mm is dipped vertically into the liquid through the airtight piston hole, as shown in the figure. The air in the container is isothermally compressed from its original volume V_0 to $\frac{100}{101}V_0$ with the movable piston. Considering air as an ideal gas, the height (h) of the liquid column in the capillary above the liquid level in cm is_______.

[Given: Surface tension of the liquid is 0.075 N m^{-1}, atmospheric pressure is 10^5 N m^{-2}, acceleration due to gravity (g) is 10 m s^{-2}, density of the liquid is 10^3 kg m^{-3} and contact angle of capillary surface with the liquid is zero]



[:ANS] **25**

[:SOLN]



For isothermal process

$$P_0V_0 = P \times \frac{100}{101} V_0$$

$$P = \frac{101}{100} P_0$$

Now,

$$P_0 - \frac{2T}{R} + \rho g h = \frac{101 P_0}{100}$$

$$h = \frac{2T}{\rho g R} + \frac{P_0}{100 \rho g}$$

$$= \frac{2 \times 0.75}{10^3 \times 10 \times 0.1 \times 10^{-3}} + \frac{10^5}{100 \times 10^3 \times 10}$$

$$= 0.25 \text{m}$$

$$= 25 \text{cm}$$

[:Q.12] In a radioactive decay process, the activity is defined as $A = -\frac{dN}{dt}$, where $N(t)$ is the number of radioactive nuclei at time t. Two radioactive sources, S_1 and S_2 have same activity at time t = 0. At a later time, the activities of S_1 and S_2 are A_1 and A_2, respectively. When S_1 and S_2 have just completed their 3rd and 7th half-lives, respectively, the ratio A_1/A_2 is __________.

[:ANS] **16**

[:SOLN] $A = A_0 e^{-\lambda t}$

$$\frac{A_1}{A_2} = \frac{A_0 e^{-\lambda_1 t_1}}{A_0 e^{-\lambda_2 t_2}}$$

$$= \frac{e^{-\lambda_1 t_1}}{e^{-\lambda_2 t_2}}$$

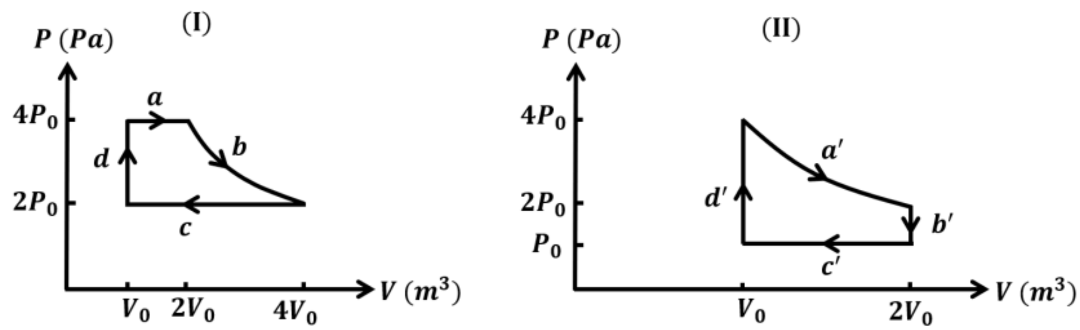
$$t_1 = 3(T_1)_{1/2}$$

$t_1 = 3 \times \frac{\ell n2}{\lambda_1}$

$\lambda_1 t_2 = 3\ell n2$ and $\lambda_2 t_2 = 7\ell n2$

$$= \frac{e^{-3\ell n2}}{e^{-7\ell n2}} = \frac{\left(\frac{1}{2^3}\right)}{\left(\frac{1}{2^7}\right)} = 2^4 = 16$$

[:Q.13] One mole of an ideal gas undergoes two different cyclic processes I and II, as shown in the P-V diagrams below. In cycle I, processes a, b, c and d are isobaric, isothermal, isobaric and isochoric, respectively. In cycle II, processes a', b', c' and d' are isothermal, isochoric, isobaric and isochoric, respectively. The total work done during cycle I is W_I and that during cycle II is W_{II}. The ratio W_I/W_{II} is _______.



[:ANS] **2**

[:SOLN] $W_I = W_a + W_b + W_c + W_d$

$= 4P_0V_0 + 8P_0V_0\ell n2 - 6P_0V_0 + 0$

$W_I = 2P_0V_0(4\ell n2 - 1)$

$W_{II} = W_{a'} + W_{b'} + W_{c'} + W_{d'}$

$= 4P_0V_0\ell n2 + 0 - P_0V_0 + 0$

$W_{II} = P_0V_0(4\ell n2 - 1)$

$$\frac{W_I}{W_{II}} = \frac{2P_0V_0(4\ell n2 - 1)}{P_0V_0(4\ell n2 - 1)} = 2$$

Ans. 2

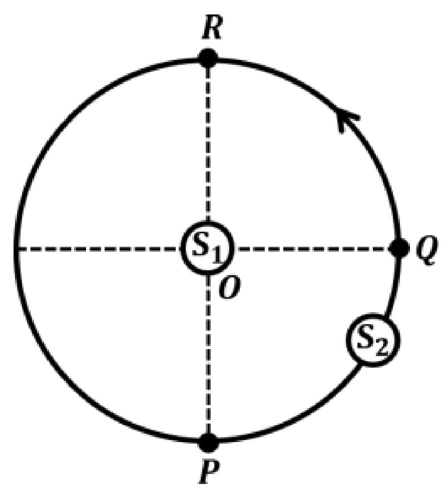
SECTION 4 (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

PARAGRAPH - I

S_1 and S_2 are two identical sound sources of frequency 656 Hz. The source S_1 is located at O and S_2 moves anti-clockwise with a uniform speed $4\sqrt{2}\,\text{m s}^{-1}$ on a circular path around O, as shown in the figure. There are three points P, Q and R on this path such that P and R are diametrically opposite while Q is equidistant from them. A sound detector is placed at point P. The source S_1 can move along direction OP.

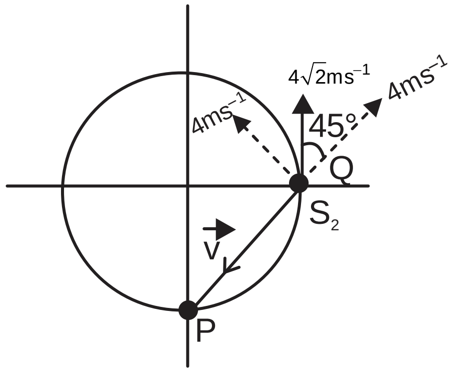
[Given: The speed of sound in air is 324 m s^{-1}]



[:Q.14] When only S_2 is emitting sound and it is at Q, the frequency of sound measured by the detector in Hz is _________.

[:ANS] **648 Hz**

[:SOLN]



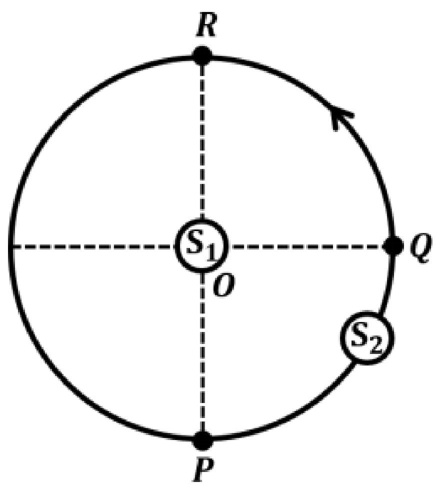
$$f = \frac{v}{v + 4\sqrt{2}\cos 45^\circ} 656$$

$$= \frac{324}{328} \times 656 = 648\,\text{Hz}$$

PARAGRAPH - I

S_1 and S_2 are two identical sound sources of frequency 656 Hz. The source S_1 is located at O and S_2 moves anti-clockwise with a uniform speed $4\sqrt{2}\,\text{m s}^{-1}$ on a circular path around O, as shown in the figure. There are three points P, Q and R on this path such that P and R are diametrically opposite while Q is equidistant from them. A sound detector is placed at point P. The source S_1 can move along direction OP.

[Given: The speed of sound in air is 324 m s^{-1}]



[:Q.15] Consider both sources emitting sound. When S_2 is at R and S_1 approaches the detector with a speed 4 m s^{-1}, the beat frequency measured by the detector is _______Hz.

[:ANS] **8.20**

[:SOLN] Frequency registered from S_2 (when it at R) by detector, f_1 = 656 Hz

Frequency registered from S_1 by detector, $f_2 = \frac{324}{320} \times 656\,\text{Hz}$

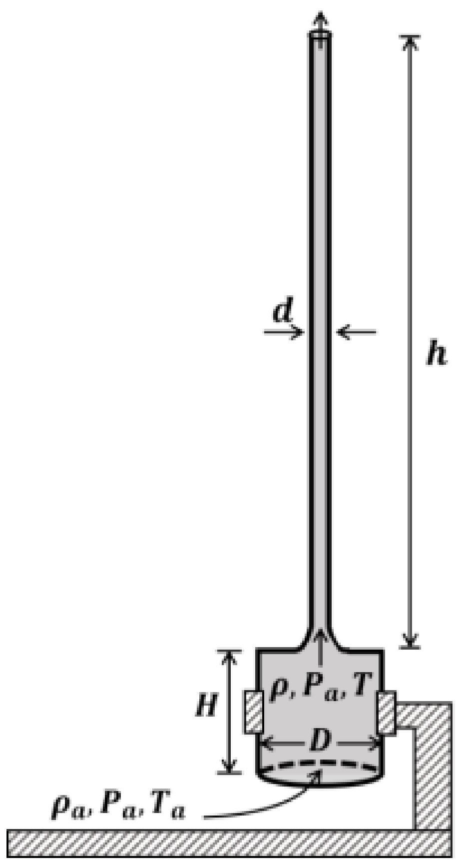
Beat frequency $= f_2 - f_1$

$= \frac{4}{320} \times 656 = 8.2\,\text{Hz}$

PARAGRAPH - II

A cylindrical furnace has height (H) and diameter (D) both 1 m. It is maintained at temperature 360 K. The air gets heated inside the furnace at constant pressure P_a and its temperature becomes T = 360 K. The hot air with density ρ rises up a vertical chimney of diameter d = 0.1 m and height h = 9 m above the furnace and exits the chimney (see the figure). As a result, atmospheric air of density ρ_a = 1.2 kg m^{-3}, pressure P_a and temperature T_a = 300 K enters the furnace. Assume air as an ideal gas, neglect the variations in ρ and T inside the chimney and the furnace. Also ignore the viscous effects.

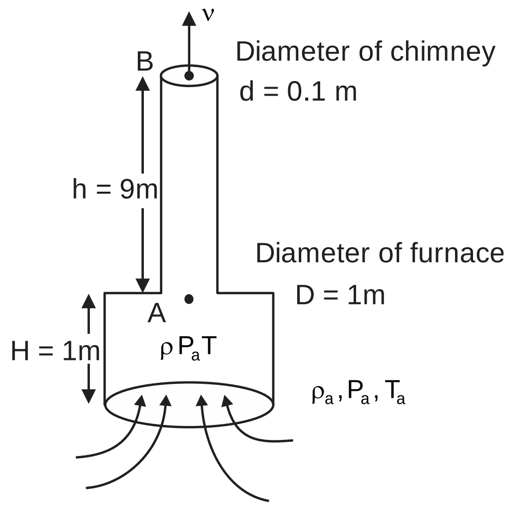
[Given: The acceleration due to gravity g = 10 m s^{-2} and π = 3.14]



[:Q.16] Considering the air flow to be streamline, the steady mass flow rate of air exiting the chimney is _______ gm s^{-1}.

[:ANS] **60.80**

[:SOLN]



Using ideal gas equation $P = \rho \frac{RT}{M}$

$\therefore$ here $\rho_a T_a = \rho T$

$\Rightarrow$ $\rho = 1 kg/m^3$

Using Bernoulli's theorem :

$$\frac{1}{2}\rho v_B^2 + P_B + \rho gh = \frac{1}{2}\rho v_A^2 + P_A$$

(Assume that the density of air remains almost same at the time of exit from chimney as that is inside chimney)

$$\frac{1}{2}\rho v_B^2 - \frac{1}{2}\rho v_A^2 = P_A - P_B - \rho gh$$

$$\Rightarrow \quad \frac{1}{2}\rho v_B^2\left(1 - \frac{v_A^2}{v_B^2}\right) = P_a - \{P_a - \rho_a g(H+h)\} - \rho gh$$

$$\Rightarrow \quad \frac{1}{2}\rho v_B^2\left(1 - \frac{A_B^2}{A_A^2}\right) = \rho_a g(H+h) - \rho gh$$

$$\Rightarrow \quad v_B^2 = 2\left[\frac{\rho_a}{\rho}(H+h)g - gh\right] \qquad \left[\because \frac{A_B^2}{A_A^2} <<< 1\right]$$

$$\Rightarrow \quad v_B = \sqrt{2\left[\frac{\rho_a}{\rho}(H+h)g - gh\right]}$$

$$= \sqrt{2(120-90)}$$

$$= \sqrt{60}\ \text{ms}^{-1}$$

Mass rate of flow of air

$$= \rho v_B\left(\frac{\pi d^2}{4}\right) = 6.080 \times 10^{-2}\ \text{kg/s}$$

$$= 60.80\ \text{g/s}$$

PARAGRAPH - II

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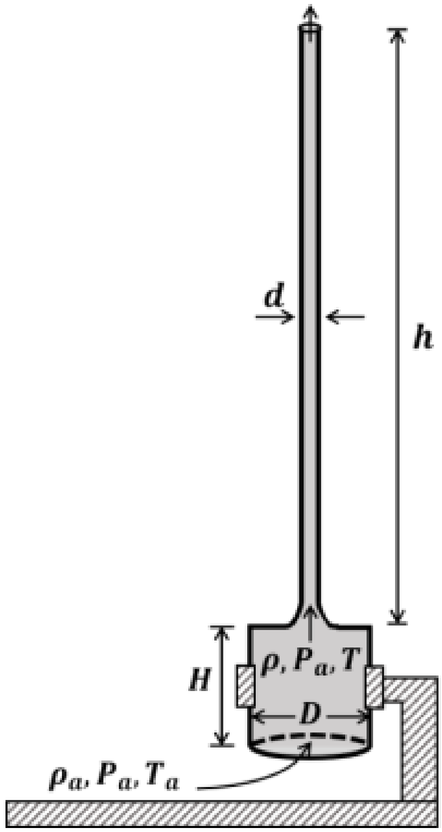
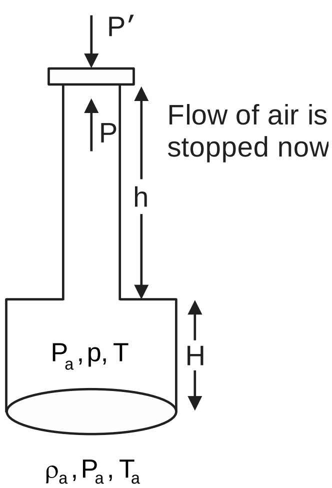
[Given: The acceleration due to gravity g = 10 m s^{-2} and π = 3.14]



[:Q.17] When the chimney is closed using a cap at the top, a pressure difference ΔP develops between the top and the bottom surfaces of the cap. If the changes in the temperature and density of the hot air, due to the stoppage of air flow, are negligible then the value of ΔP is ______ N m^{-2}.

[:ANS] **20**

[:SOLN]



$$\Delta P = |P' - P|$$

$$= \left| [P_a - \rho_a g(H+h)] - [P_a - \rho g(H+h)] \right|$$

$$= \left| [(\rho - \rho_a) g(H+h)] \right|$$

$$= 20 \text{ Pascal}$$