JEE (ADVANCED) 2023 PAPER-1

[PAPER WITH SOLUTION]

HELD ON SUNDAY 04TH JUNE 2023

MATHEMATICS

SECTION1 (Maximum Marks : 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s)is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;

Zero Marks :0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 mark;

choosing ONLY (B) will get +1 mark;

choosing ONLY (D) will get +1 mark;

choosing no option (i.e. the question is unanswered) will get 0 marks; and

choosing any other combination of options will get −2 marks.

[:Q.1] Let S = (0,1) ∪ (1,2) ∪ (3,4) and T = {0,1, 2,3} . Then which of the following statements is(are) true?

[:A] There are infinitely many functions from S to T

[:B] There are infinitely many strictly increasing functions from S to T

[:C] The number of continuous functions from S to T is at most 120

[:D] Every continuous function from S to T is differentiable

[:ANS] A, C, D

[:SOLN] There are infinitely many elements in the domain 5 and 4 elements in co-domain T, therefore, there will be infinitely many functions from S to T

Further, for a function from S to T to be continuous it should take a single value in (0, 1) then a single value in (1, 2) then a single value in (3, 4), therefore, the number of continuous functions is 4 × 4 × 4 = 64

and every such continuous function from S to T will also be differentiable.

[:Q.2] Let T_1 and T_2 be two distinct common tangents to the ellipse E : $\frac{x^2}{6}+\frac{y^2}{3}=1$ and the parabola P : $y^2 = 12x$. Suppose that the tangent T_1 touches P and E at the points A_1 and A_2, respectively and the tangent T_2 touches P and E at the points A_4 and A_3, respectively. Then which of the following statements is(are) true?

[:A] The area of the quadrilateral $A_1A_2A_3$ A_4 is 35 square units

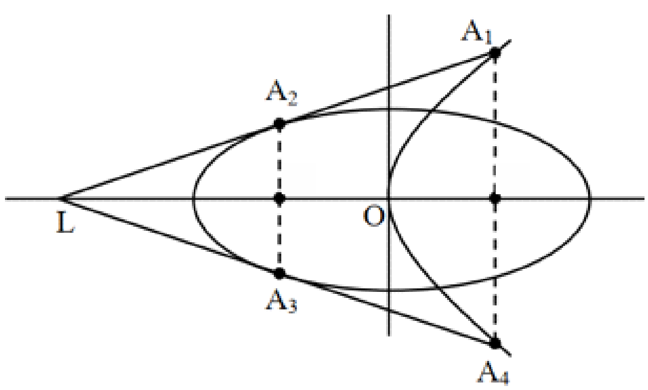
[:B] The area of the quadrilateral $A1A_2A_3$ A_4 is 36 square units

[:C] The tangents T_1 and T_2 meet the x -axis at the point (–3,0)

[:D] The tangents T_1 and T_2 meet the x -axis at the point (–6,0)

[:ANS] A, C

[:SOLN]



P : $y^2 = 12x$, E : $\frac{x^2}{6}+\frac{y^2}{3}=1$

Let equation of tangent to P be $y = mx + \frac{3}{m}$

for common tangent to P & E

$\frac{3}{m} = \pm\sqrt{6m^2 + 3}$

$\frac{9}{m^2} = 6m^2 + 3 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1$

so common tangents are y = x + 3, y = –x – 3

Area of quadrilateral A1A2A3A4

$= \frac{1}{2}(2+12)\times 5$

= 35

[:Q.3] Let f :[0,1] → [0,1] be the function defined by $f(x) = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$. Consider the square region S = [0,1] × [0,1]. Let G = {(x, y) ∈ S : y > f(x)} be called the green region and R = {(x, y) ∈ S : y < f(x)} be called the red region. Let L_h = {(x, h) ∈ S : x ∈ [0,1]} be the horizontal line drawn at a height h∈[0,1]. Then which of the following statements is(are) true?

[:A] There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the green region below the line L_h

[:B] There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the red region below the line L_h

[:C] There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the red region below the line L_h

[:D] There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the green region below the line L_h

[:ANS] B,C,D

[:SOLN] $f(x) = \frac{x^3}{3} - x^2 + \frac{5x}{9} + \frac{17}{36}$

$f'(x) = x^2 - 2x + \frac{5}{9}$

$f'(x) = 0$ at $x = \frac{1}{3}$ in [0, 1]

A_R = Area of Red region

A_G = Area of Green region

$$A_R = \int_0^1 f(x)dx = \frac{1}{2}$$

Total area = 1

$$\Rightarrow A_G = \frac{1}{2}$$

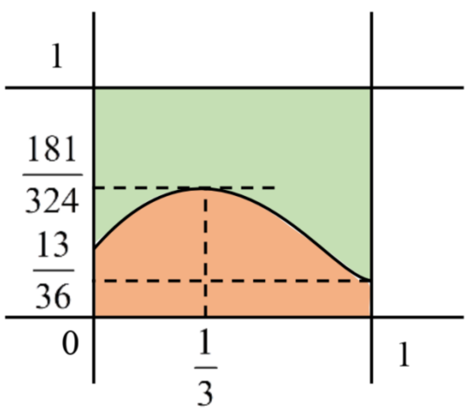
$$\int_0^1 f(x)dx = \frac{1}{2}$$

$A_G = A_R$

$$f(0) = \frac{17}{36}$$

$$f(1) = \frac{13}{36}$$

$$f\left(\frac{1}{3}\right) = \frac{181}{324}$$



(A) Correct when $h = \frac{3}{4}$ but $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$

$\Rightarrow$ (A) is incorrect

(B) Correct when h = 1/4

$\Rightarrow$ (B) is correct

(C) When $h = \frac{181}{324}$, $A_R = \frac{1}{2}$, $A_G < \frac{1}{2}$

$h = \frac{13}{36}, A_R < \frac{1}{2}, A_G = \frac{1}{2}$

$\Rightarrow$ $A_R = A_G$ for some $h \in \left(\frac{13}{36}, \frac{181}{324}\right)$

$\Rightarrow$ (C) is correct

(D) Option (D) is remaining coloured part of option (C), hence option (D) is also correct.

SECTION 2 (Maximum Marks : 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

[:Q.4] Let f : (0, 1) → R be the function defined as f(x) = $\sqrt{n}$ if $x \in \left[\frac{1}{n+1}, \frac{1}{n}\right)$ where $n \in N$. Let g : (0, 1) → R be a function such that $\int_{x^2}^{x} \sqrt{\frac{1-t}{t}}\,dt < g(x) < 2\sqrt{x}$ for all $x \in (0, 1)$. then $\lim_{x\to 0} f(x)g(x)$

[:A] does NOT exist

[:B] is equal to 1

[:C] is equal to 2

[:D] is equal to 3

[:ANS] C

[:SOLN] $f(x) = \frac{1}{\sqrt{x}}$ as $x \to 0^+$, because $\frac{1}{n} \to \infty$

RHL : $\lim_{x\to 0} f(x)g(x) = \lim_{x\to 0} \frac{1}{\sqrt{x}} \times 2\sqrt{x} = 2$

LHL : $\lim_{x\to 0} f(x)\cdot g(x) = \lim_{x\to 0} \frac{1}{\sqrt{x}} \cdot \int_{x^2}^{x} \sqrt{\frac{1-t}{t}}\,dt$

using L-H Rule,

$$\lim_{x\to 0} \frac{\sqrt{\frac{1-x}{x}} - \sqrt{\frac{1-x^2}{x^2}} \times 2x}{1/2\sqrt{x}} = \lim_{x\to 0} 2\sqrt{x}\left\{\sqrt{\frac{1-x}{x}} - 2\sqrt{1-x^2}\right\}$$

using sandwich theorem,

$\lim_{x \to 0} f(x) \cdot g(x) = 2$

[:Q.5] Let Q be the cube with the set of vertices $\{(x_1, x_2, x_3) \in R^3 : x_1, x_2, x_3 \in \{0, 1\}\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0,0,0) and (1,1,1) is in S. For lines ℓ_1 and ℓ_2, let $d(\ell_1, \ell_2)$ denote the shortest distance between them. Then the maximum value of $d(\ell_1, \ell_2)$, as ℓ_1 varies over F and ℓ_2 varies over S , is

[:A] $\frac{1}{\sqrt{6}}$

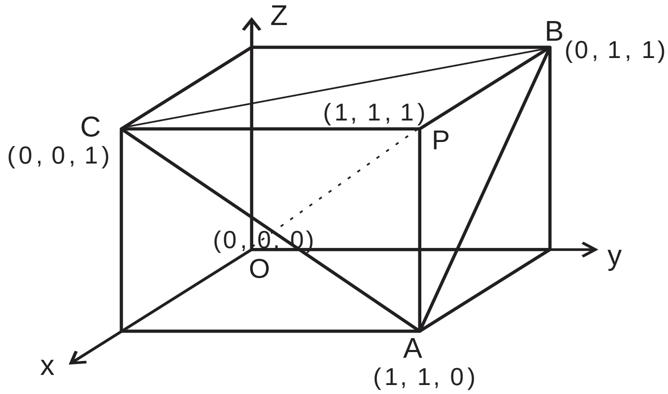
[:B] $\frac{1}{\sqrt{8}}$

[:C] $\frac{1}{\sqrt{3}}$

[:D] $\frac{1}{\sqrt{12}}$

[:ANS] A

[:SOLN]



OP: $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$

AB : $\frac{x-1}{-1} = \frac{y-1}{0} = \frac{z}{1}$

d(OP, AB) = 1 $\frac{(\hat{i}+\hat{j})\cdot\{(\hat{i}+\hat{j}+\hat{k})\times(-\hat{i}+\hat{k})\}}{|(\hat{i}+\hat{j}+\hat{k})\times(-\hat{i}+\hat{k})|} = \frac{1}{\sqrt{6}}$

[:Q.6] Let X = $\left\{(x,y)\in Z\times Z:\frac{x^2}{8}+\frac{y^2}{20}<1 \text{ and } y^2<5x\right\}$. Three distinct points P, Q and R are randomly chosen from X. Then the probability that P, Q and R form a triangle whose area is a positive integer, is

[:A] $\frac{71}{220}$

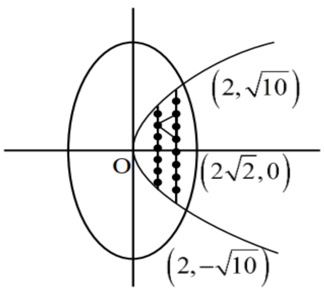
[:B] $\frac{73}{220}$

[:C] $\frac{79}{220}$

[:D] $\frac{83}{220}$

[:ANS] B

[:SOLN]



Required ways to make

Area an Integer is (5 + 3 + 1) × 5 + (3 + 1) × 7 = 73

Total ways = $^{12}C_3$ = 220

$\therefore$ Probability = $\frac{73}{220}$

[:Q.7] Let P be a point on the parabola $y^2 = 4ax$, where a > 0. The normal to the parabola at P meets the x -axis at a point Q. The area of the triangle PFQ, where F is the focus of the parabola, is 120. If the slope m of the normal and a are both positive integers, then the pair (a, m) is

[:A] (2,3)

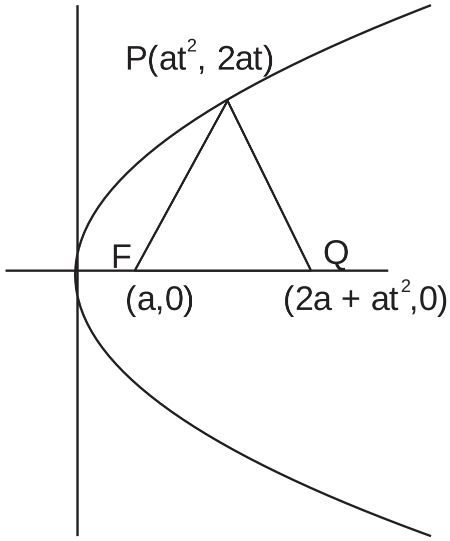
[:B] (1,3)

[:C] (2,4)

[:D] (3,4)

[:ANS] A

[:SOLN]



Area (ΔPFQ)

$\Rightarrow \frac{1}{2}\times\left(a+at^2\right)\times 2at=120$

$\Rightarrow$ at (a + at^2) = 120

$\Rightarrow$ a^2(t(1 + t^2)) = 120

$\therefore$ a = 2, b = 3

SECTION 3 (Maximum Marks : 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If ONLY the correct integer is entered;

  Zero Marks : 0 In all other cases.

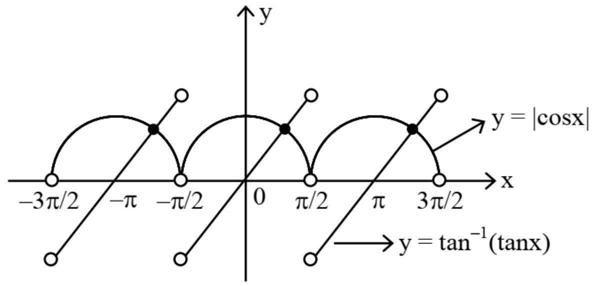
[:Q.8] Let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, for $x \in R$. Then the number of real solutions of the equation $\sqrt{1+\cos(2x)} = \sqrt{2}\tan^{-1}(\tan x)$ in the set $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right)\cup\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\cup\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is equal to

[:ANS] 3

[:SOLN] $\sqrt{1+\cos 2x} = \sqrt{2}\tan^{-1}(\tan x)$

$\sqrt{2}\left|\cos x\right| = \sqrt{2}\tan^{-1}(\tan x)$

$\left|\cos x\right| = \tan^{-1}(\tan x)$



No of solution = 3

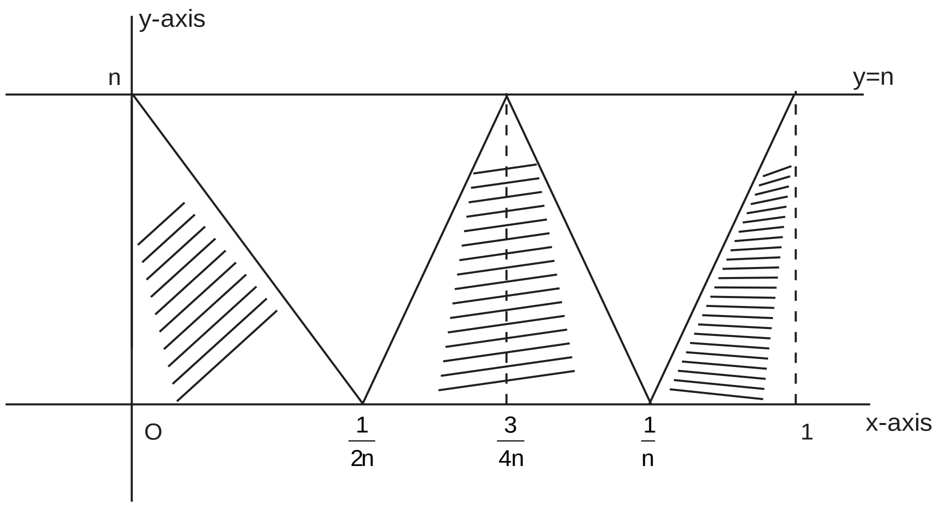
[:Q.9] Let $n \geq 2$ be a natural number and $f : [0, 1] \to R$ be the function defined by

$$f(x)=\begin{cases} n(1-2nx) & \text{if } 0 \leq x \leq \frac{1}{2n} \\ 2n(2nx-1) & \text{if } \frac{1}{2n} \leq x \leq \frac{3}{4n} \\ 4n(1-nx) & \text{if } \frac{3}{4n} \leq x \leq \frac{1}{n} \\ \frac{n}{n-1}(nx-1) & \text{if } \frac{1}{n} \leq x \leq 1 \end{cases}$$

If n is such that the area of the region bounded by the curves x = 0 , x = 1 , y = 0 and y = f(x) is 4 , then the maximum value of the function f is

[:ANS] 8

[:SOLN]



$$\text{Area} = \frac{1}{2}\times\frac{1}{2n}\times n+\frac{1}{2}\left(\frac{1}{n}-\frac{1}{2n}\right)\times n+\frac{1}{2}\left(1-\frac{1}{n}\right)\times n$$

$$= \frac{1}{4}+\frac{1}{4}+\frac{1}{2}(n-1)$$

given $\quad \frac{1}{2}+\frac{1}{2}(n-1)=4$

$1 + n - 1 = 8 \Rightarrow \boxed{n = 8}$

Thus maximum value of function f is 8

[:Q.10] Let $7\overbrace{5\cdots\cdots5}^{r}7$ denote the (r + 2) digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum S = 77 + 757 + 7557 + ······+ $7\overbrace{5\cdots\cdots5}^{98}7$ .If $S = \dfrac{7\overbrace{5\cdots\cdots5}^{99}7 + m}{n}$, where m and n are natural numbers less than 3000, then the value of m + n is

[:ANS] 1219

[:SOLN] $S = 77 + 757 + 7557 + \ldots\ldots + 7\overbrace{5\cdots\cdots5}^{98}7$

$= 7 \times 10 + 7 \times 10^2 + 7 \times 10^3 + \ldots\ldots + 7 \times 10^{99} + 7 \times 99$

$+ 50\,(1 + 11 + 111 + \ldots\ldots \text{ up 98 terms})$

$= \dfrac{7\left(10^{100} - 10\right)}{9} + \dfrac{50}{9}\,(10 + 10^2 + \ldots. + \text{upto 98 terms} - 98) + 693$

$= \dfrac{7\left(10^{100} - 10\right)}{9} + \dfrac{50}{9}\left(\dfrac{10\left(10^{98} - 1\right)}{9} - 98\right) + 693$

$= \dfrac{7\left(10^{100} - 10\right)}{9} + \dfrac{50}{9}\left(\dfrac{10\left(10^{99} - 10\right)}{9} - 98\right) + 693$

$= \dfrac{7\left(10^{100} - 10\right)}{9} + \dfrac{50}{9}\left(\dfrac{10^{99} - 1}{9} - 99\right) + 693$

$= \dfrac{1}{9}\left(7 \times 10^{100} + 50\left(\dfrac{10^{99} - 1}{9}\right) + 7\right) - \dfrac{7}{9} - \dfrac{70}{9} - \dfrac{50 \times 99}{9} + 693$

$= \dfrac{1}{9}\left(7\overbrace{5\cdots\cdots5}^{99}7\right) + \dfrac{1210}{9}$

$= \dfrac{S = 7\overbrace{5\cdots\cdots5}^{99}7 + 1210}{9} = \dfrac{7\overbrace{5\cdots\cdots5}^{99}7 + m}{n}$

m = 1210, n = 9

m + n = 1219

[:Q.11] Let $A=\left\{\frac{1967+1686i\sin\theta}{7-3i\cos\theta}:\theta\in R\right\}$. If A contains exactly one positive integer n, then the value of n is

[:ANS] 281

[:SOLN] $A=\frac{1967+1686i\ \sin\theta}{7-3i\cos\theta}$

$A=\frac{1967+1686i\sin\theta}{7-3i\cos\theta}\times\frac{7+3i\cos\theta}{7+3i\cos\theta}$ is an integer

$\Rightarrow$ 1967 × cosθ + 7 × 1686 sin θ = 0

$\Rightarrow \tan\theta = \frac{-1967\times 3}{7\times 1686}=\frac{-5901}{11802}=-\frac{1}{2}$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{5}}$ & $\cos\theta = -\frac{2}{\sqrt{5}}$

Thus $A = \frac{1967\times 7-3\times 1686\sin\theta\cos\theta}{49+9\cos^2\theta}$

$=\frac{1967\times 7+3\times 1686\times\frac{2}{5}}{49+9\times\frac{4}{5}}$

= 281

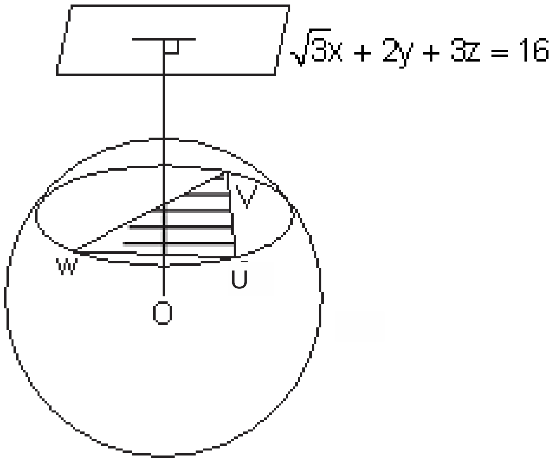
[:Q.12] Let P be the plane $\sqrt{3}x + 2y + 3z = 16$ and let

$S=\left\{\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k} : \alpha^2+\beta^2+\gamma^2=1 \text{ and the distance of } (\alpha, \beta, \gamma) \text{ from the plane P is } \frac{7}{2}\right\}$

Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$. Let V be the volume of the parallelepiped determined by vectors $\vec{u}, \vec{v}$ and $\vec{w}$. Then the value of $\frac{80}{\sqrt{3}}V$ is

[:ANS] 45

[:SOLN]



Point (α, β, γ) lies on the sphere with center O(0, 0, 0)

and radius 1.

Distance of plane P from 0

$$OP = \frac{16}{\sqrt{3+4+9}} = 4$$

$\vec{u}, \vec{v}, \vec{w}$ in S can be taken as position vectors of points, U, V, W as shown, as

$$|\vec{u}-\vec{v}| = |\vec{v}-\vec{w}| = |\vec{w}-\vec{u}|$$

implies Point U, V, W are the vertices of an equilateral triangle inscribed in a circle of radius=

$$\sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$$

Now, volume of parallelopiped V = 6 × volume of tetrahedron OUVW

$$= 6 \times \frac{1}{3} \text{ Area of } \Delta UVW \times \text{height}$$

$$= 6 \times \frac{1}{3} \times \frac{3\sqrt{3}}{4} \times \left(\frac{\sqrt{3}}{2}\right)^2 \times \frac{1}{2}$$

$$V = \frac{9}{16}\sqrt{3}$$

$$\therefore \frac{80}{\sqrt{3}} V = \frac{80}{\sqrt{3}} \times \frac{9}{16}\sqrt{3} = 45.$$

[:Q.13] Let a and b be two nonzero real numbers. If the coefficient of x^5 in the expansion of $\left(ax^2 + \frac{70}{27bx}\right)^4$ is equal to the coefficient of x^{-5} in the expansion of $\left(ax - \frac{1}{bx^2}\right)^7$, then the value of 2b is

[:ANS] 3

[:SOLN] Coefficient of x5 in $\left(ax^2 + \frac{70}{27bx}\right)^4$

$T_{r+1} = {}^4C_r\,(ax^2)^{4-r}\left(\frac{70}{27bx}\right)^r$

$= {}^4C_r\,a^{4-r}\,x^{8-2r} \times \frac{70^r}{(27b)^r\,x^r}$

$= {}^4C_r\,\frac{a^{4-r}\cdot 70^r}{(27b)^r} \times x^{8-3r}$

for Coefficient of x^5 $\quad 8 - 3r = 5 \quad \Rightarrow \quad r = 1$

so Coefficient of x^5 is ${}^4C_1\,\frac{a^3 \times 70}{27b}$

for Coefficient of x^{-5} in $\left(ax - \frac{1}{bx^2}\right)^7$

$T'_{r+1} = {}^7C_r\,(ax)^{7-r}\left(-\frac{1}{bx^2}\right)^r$

$= {}^7C_r\,a^{7-r}\,x^{7-r}\,(-1)^r\,\frac{1}{b^r x^{2r}}$

$= {}^7C_r\,\frac{a^{7-r}(-1)^r}{b^r}\;x^{7-3r}$

for coefficient of x^{-5} $\quad 7 - 3r = -5 \Rightarrow r = 4$

so coefficient of x^{-5} is ${}^7C_4\,\frac{a^3}{b^4}$

given $\quad \frac{4 \times \cancel{a^3} \times \cancel{70}^{\,2}}{27b} = \frac{\cancel{7} \times \cancel{6} \times 5}{\cancel{3 \times 2} \times 1} \times \frac{\cancel{a^3}}{b^4}$

$b^3 = \frac{27}{8} \Rightarrow b = \frac{3}{2}$

$\Rightarrow 2b = 3 \qquad$ Ans. = 3

SECTION 4 (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases

[:Q.14] Let α, β and γ be real numbers. Consider the following system of linear equations

$x + 2y + z = 7$

$x + az = 11$

$2x - 3y + \beta z = \gamma$

Match each entry in **List-I** to the correct entries in **List-II**.

LIST-I	**LIST-II**
(P) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$, then the system has	(1) a unique solutions
(Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has	(2) no solutions
(R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ Where $\alpha = 1$ and $\gamma \neq 28$, then the system has	(3) infinitely many solutions
(S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has as a solutions	(4) $x = 11, y = -2$ and $z = 0$
	(5) $x = -15, y = 4$ and $z = 0$ as a solutions

Codes :

The correct option is:

(A) (P) → (3) (Q) → (2) (R) → (1) (S) → (4)

(B) (P) → (3) (Q) → (2) (R) → (5) (S) → (4)

(C) (P) → (2) (Q) → (1) (R) → (4) (S) → (5)

(D) (P) → (2) (Q) → (1) (R) → (1) (S) → (3)

[:ANS] A

[:SOLN] $x + 2y + z = 7$ ....(1)

$x + \alpha z = 11$ ________(2)

$2x - 3y + \beta z = \gamma$ ______(3)

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix}$$

$= 3\alpha - 2(\beta - 2\alpha) + (-3)$

$= 3\alpha - 2\beta + 4\alpha - 3$

$\Delta = 7\alpha - 2\beta - 3$ ____

$$\Delta_1 = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & \alpha \\ r & -3 & \beta \end{vmatrix}$$

$= 21\alpha - 2(11\beta - \alpha\gamma) + (-33)$

$\Delta_1 = 21\alpha - 22\beta + 2\alpha\gamma - 33$ ______

$$\Delta_2 = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & r & \beta \end{vmatrix} = (11\beta - \alpha\gamma) - 7(\beta - 2\alpha) + (\gamma - 22)$$

$\Delta_2 = 4\beta - \alpha\gamma + 14\alpha + \gamma - 22$ _______

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & r \end{vmatrix} = 33 - 2(\gamma - 22) + 7(-3)$$

$\Delta_3 = 33 - 2\gamma + 44 - 21$

$= 77 - 21 - 2\gamma$

$\Delta_3 = (56 - 2\gamma)$ ________

(P) at $\beta = \frac{1}{2}(7\alpha - 3)$ $\quad \Delta = 0$

at $\gamma = 28$ $\quad \Delta_3 = 0$

$\Delta_1 = 21\alpha - \frac{22}{2}(7\alpha - 3) + 56\alpha - 33$

$\Delta_1 = 21\alpha - 77\alpha + 33 + 56\alpha - 33 = 0$

$\Delta_2 = 2(7\alpha - 3) - 28\alpha + 14\alpha + 6$

$= 14\alpha - 28\alpha + 14\alpha = 0$

$\because \Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

(Q) $\Delta = 0$ but $\Delta_3 \neq 0$ so can't be unique solution so (c) & (D) Not possible.

Now we have to choose option from (A) & (B) So P $\to$ (3) & Q $\to$ (2) Now for (R) $\Delta \neq 0$ so unique soln.

now $\alpha = 1$, $\gamma \neq 28$, $\beta \neq 2$

$\because z = \frac{\Delta_3}{\Delta} \neq 0$ so (5) option is incorrect

so R $\to$ (1)

A Correct

[:Q.15] Consider the given data with frequency distribution

x_i	3	8	11	10	5	4
f_i	5	2	3	2	4	4

Match each entry in List-I to the correct entries in List-II.

LIST-I	**LIST-II**
(P) The mean of the above data is	(1) 2.5
(Q) The median of the above data is	(2) 5
(R) The mean deviation about the mean of the above data is	(3) 6
(S) The mean deviation about the median of the above data is	(4) 2.7
	(5) 2.4

Codes :

The correct option is:

(A) (P) $\to$ (3) (Q) $\to$(2) (R) $\to$ (4) (S) $\to$ (5)

(B) (P) $\to$ (3) (Q) $\to$(2) (R) $\to$ (1) (S) $\to$ (5)

(C) (P) $\to$ (2) (Q) $\to$(3) (R) $\to$ (4) (S) $\to$ (1)

(D) (P) $\to$ (3) (Q) $\to$(3) (R) $\to$ (5) (S) $\to$ (5)

[:ANS] A

[:SOLN] (P) $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{3\times5+8\times2+11\times3+10\times2+5\times4+4\times4}{20} = 6$

(Q) $\frac{N}{2} = \frac{20}{2} = 10$

Median = Mean of 10^{th} and 11^{th} Observation

$= \frac{5+5}{2} = 5$

x_i	3	4	5	8	10	11
f_i	5	4	4	2	2	3

(R) mean deviation about Mean = $\frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$

$= \frac{5\times 3 + 4\times 2 + 4\times 1 + 2\times 2 + 2\times 4 + 3\times 5}{20} = 2.7$

(S) mean deviation about median = $\frac{\sum f_i |x_i - M|}{\sum f_i}$

$= \frac{5\times 2 + 4\times 1 + 4\times 0 + 2\times 3 + 2\times 5 + 3\times 6}{20} = 2.4$

(P) $\to$ (3) (Q) $\to$ (2) (R) $\to$ (4) (S) $\to$ (5)

[:Q.16] Let ℓ_1 and ℓ_2 be the lines $\vec{r}_1 = \lambda(\hat{i}+\hat{j}+\hat{k})$ and $\vec{r}_2 = (\hat{j}-\hat{k}) + \mu(\hat{i}+\hat{k})$, respectively. Let X be the set of all the planes H that contain the line ℓ_1. For a plane H, let d(H) denote the smallest possible distance between the points of ℓ_2 and H. Let H_0 be a plane in X for which $d(H_0)$ is the maximum value of d(H) as H varies over all planes in X.

Match each entry in List-I to the correct entries in List-II.

LIST-I	**LIST-II**
(P) the value of $d(H_0)$ is	(1) $\sqrt{3}$
(Q) The distance of the point (0, 1, 2) from H_0 is	(2) $\frac{1}{\sqrt{3}}$
(R) The distance of origin from H_0 is	(3) 0
(S) The distance of origin from the point of intersection of planes y = z, x = 1 and H_0 is	(4) $\sqrt{2}$
	(5) $\frac{1}{\sqrt{2}}$

The correct options is :

Codes :

(A) (P) $\to$ (2) (Q) $\to$ (4) (R) $\to$ (5) (S) $\to$ (1)

(B) (P) $\to$ (5) (Q) $\to$ (4) (R) $\to$ (3) (S) $\to$ (1)

(C) (P) $\to$ (2) (Q) $\to$ (1) (R) $\to$ (3) (S) $\to$ (2)

(D) (P) $\to$ (5) (Q) $\to$ (1) (R) $\to$ (4) (S) $\to$ (2)

[:ANS] B

[:SOLN] H is perpendicular to $\lambda(\hat{i}+\hat{j}+k)$ and also $\mu(\hat{i}+k)$

$\therefore$ Their common perpendicular is $\begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i}-k$

So, equation of plane is x – z = 0

(P) $d(H_0) = \frac{1}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$

(Q) $\frac{0+2}{\sqrt{2}} = \sqrt{2}$

(R) $\frac{0}{\sqrt{2}} = 0$

(S) Point of intersection of y = z , x = 1 and H_0 is (1,1,1)

$\therefore$ distance $= \sqrt{3}$

Ans. (B)

[:Q.17] Let z be a complex number satisfying $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$, where $\bar{z}$ denotes the complex conjugate of z . Let the imaginary part of z be nonzero.

Match each entry in List-I to the correct entries in List-II.

LIST-I	LIST-II
(P) $|z|^2$ is equal to	(1) 12
(Q) $|z-\bar{z}|^2$ is equal to	(2) 4
(R) $|z|^2 + |z+\bar{z}|^2$ is equal to	(3) 8
(S) $|z+1|^2$ is equal to	(4) 10
	(5) 7

Codes :

The correct options is:

(A) (P) →(1) (Q) →(3) (R)→(5) (S) →(4)
(B) (P) →(2) (Q) →(1) (R)→(3) (S) →(5)
(C) (P) →(2) (Q) →(4) (R)→(5) (S) →(1)
(D) (P) →(2) (Q) →(3) (R)→(5) (S) →(4)

[:ANS] B

[:SOLN]

$|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$ ....(i)

$|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$ ......(ii)

(i) – (ii)

$2\left(z^2 - \bar{z}^2\right) + 4\left(\bar{z} - z\right) = 0$

$\left(z - \bar{z}\right)\left(z + \bar{z}\right) - 2\left(z - \bar{z}\right) = 0$

$\left(z - \bar{z}\right)\left(z + \bar{z} - 2\right) = 0$

$z - \bar{z} \neq 0 \Rightarrow z + \bar{z} = 2$

$\Rightarrow 2x = 2 \Rightarrow x = 1$

$\Rightarrow z = 1 + iy, y \neq 0$

from (i)

$\left(1 + y^2\right)^{3/2} + 2\left(1 - y^2 + 2iy\right) + 4\left(1 - iy\right) - 8 = 0$

$\left(1 + y^2\right)^{3/2} - 2\left(1 + y^2\right) = 0$

$\left(1 + y^2\right)\left(\sqrt{1 + y^2} - 2\right) = 0$

$1 + y^2 = 4 \Rightarrow y^2 = 3$

(P) $|z|^2 = 1 + y^2 = 4$

(Q) $|z - \bar{z}|^2 = 4y^2 = 12$

(R) $|z|^2 + |z + \bar{z}|^2 = 1 + y^2 + 4 = 8$

(S) $|z + 1|^2 = 4 + y^2 = 7$