JEE (ADVANCED) 2023 PAPER-2

[PAPER WITH SOLUTION]

HELD ON SUNDAY 04TH JUNE2023

MATHEMATICS

SECTION1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is thecorrect answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If **ONLY** the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases.

[:Q.1] Let $f:[1,\infty)\to\mathbb{R}$ be a differentiable function such that $f(1)=\frac{1}{3}$ and $3\int_1^x f(t)\,dt = x\,f(x)-\frac{x^3}{3}, x\in[1,\infty)$. Let e denote the base of the natural logarithm. Then the value of f(e) is

[:A] $\frac{e^2+4}{3}$

[:B] $\frac{\log_e 4+e}{3}$

[:C] $\frac{4e^2}{3}$

[:D] $\frac{e^2-4}{3}$

[:ANS] C

[:SOLN] $3\int_1^x f(t)\,dt = x\,f(x)-\frac{x^3}{3}$

differentiating both sides w.r.t. x .
$3f(x) = f(x) + xf'(x) - x^2$
differential equation is
$2y = x\frac{dy}{dx} - x^2 \Rightarrow \frac{dy}{dx} + \left(-\frac{2}{x}\right)\cdot y = x$

Integrating Factor = $e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$

$\therefore$ Solution is $y \cdot \frac{1}{x^2} = \int \frac{1}{x}dx$

$\Rightarrow y = x^2 \ln x + cx^2$

$\Rightarrow f(1) = \frac{1}{3} \Rightarrow C = \frac{1}{3}$

$\therefore f(x) = x^2 \ln x + \frac{x^2}{3}$

$\Rightarrow f(e) = e^2 + \frac{e^2}{3} = \frac{4}{3}e^2$

Ans. (C)

[:Q.2] Consider an experiment of tossing a coin repeatedly until the outcomes of two consecutive tosses are same. If the probability of a random toss resulting in head is 1/3, then the probability that the experiment stops with head is

[:A] $\frac{1}{3}$

[:B] $\frac{5}{21}$

[:C] $\frac{4}{21}$

[:D] $\frac{2}{7}$

[:ANS] B

[:SOLN] HH $\rightarrow \frac{1}{3^2}$

THH $\rightarrow \frac{2}{3} \cdot \frac{1}{3^2}$

HTHH $\rightarrow \frac{2}{3} \cdot \frac{1}{3^3}$

THTHH $\rightarrow \left(\frac{2}{3}\right)^2 \cdot \frac{1}{3^3}$

$$\text{HTHTHH} \rightarrow \left(\frac{2}{3}\right)^2 \cdot \frac{1}{3^4}$$

$$\text{THTHTHH} \rightarrow \left(\frac{2}{3}\right)^3 \cdot \frac{1}{3^4}$$

$$\text{HTHTHTHH} \rightarrow \left(\frac{2}{3}\right)^3 \cdot \frac{1}{3^5}$$

Required Probability

$$= \frac{1}{9} + \frac{2}{3^3}\left(1+\frac{1}{3}\right) + \frac{2^2}{3^5}\left(1+\frac{1}{3}\right) + \frac{2^3}{3^7}\left(1+\frac{1}{3}\right) + \ldots\ldots + \infty$$

$$= \frac{1}{9} + \frac{4}{3}\cdot\frac{2}{3^3}\left(1+\frac{2}{3^2}+\frac{2^2}{3^4}+\ldots\ldots+\infty\right)$$

$$= \frac{1}{9} + \frac{8}{81}\cdot\frac{1}{1-\frac{2}{9}} = \frac{1}{9}+\frac{8}{81}\cdot\frac{9}{7}$$

$$= \frac{1}{9}+\frac{8}{63} = \frac{7+8}{63} = \frac{15}{63} = \frac{5}{21}$$

[:Q.3] For any $y \in \mathbb{R}$, let $\cot^{-1}(y) \in (0,\pi)$ and $\tan^{-1}(y) \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then the sum of the all the solutions of the equation $\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \cot^{-1}\left(\frac{9-y^2}{6y}\right) = \frac{2\pi}{3}$ for $0 < |y| < 3$, is equal to

[:A] $2\sqrt{3}-3$

[:B] $3-2\sqrt{3}$

[:C] $4\sqrt{3}-6$

[:D] $6-4\sqrt{3}$

[:ANS] C

[:SOLN] $-3 < y < 0 \Rightarrow \tan^{-1}\frac{6y}{9-y^2} + \pi + \tan^{-1}\frac{6y}{9-y^2} = \frac{2\pi}{3}$

$$\Rightarrow \tan^{-1}\frac{6y}{9-y^2} = -\frac{\pi}{6} \Rightarrow 6\sqrt{3}y = y^2 - 9$$

$$\Rightarrow y^2 - 6\sqrt{3}y - 9 = 0 \Rightarrow y = \frac{6\sqrt{3}-12}{2} = 3\sqrt{3}-6 \ldots\ldots\text{(i)}$$

$$0 < y < 3 \Rightarrow 2\tan^{-1}\frac{6y}{9-y^2} = \frac{2\pi}{3} \quad \Rightarrow \quad 6y = 9\sqrt{3}-\sqrt{3}y^2$$

$$\Rightarrow \sqrt{3}y^2 + 6y - 9\sqrt{3} = 0$$

$$\Rightarrow y = \frac{-6+12}{2\sqrt{3}} = \sqrt{3}\ldots\ldots\text{(ii)}$$

from (i) & (ii) sum of solutions = $3\sqrt{3}-6+\sqrt{3}=4\sqrt{3}-6$

Ans. C

[:Q.4] Let the position vectors of the points P,Q, R and S be $\vec{a}=\hat{i}+2\hat{j}-5\hat{k}$, $\vec{b}=3\hat{i}+6\hat{j}+3\hat{k}$, $\vec{c}=\frac{17}{5}\hat{i}+\frac{16}{5}\hat{j}+7\hat{k}$ and $\vec{d}=2\hat{i}+\hat{j}+\hat{k}$, respectively. Then which of the following statements is true?

[:A] The points P, Q, R and S are **NOT** coplanar

[:B] $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides PR internally in the ratio 5 : 4

[:C] $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides PR externally in the ratio 5 : 4

[:D] The square of the magnitude of the vector $\vec{b}\times\vec{d}$ is 95

[:ANS] B

[:SOLN]

$$\frac{5\left(\frac{17}{5}\hat{i}+\frac{16}{5}\hat{j}+7\hat{k}\right)+4\left(\hat{i}+2\hat{j}-5\hat{k}\right)}{5+4}=\frac{7\hat{i}+8\hat{j}+5\hat{k}}{3}=\frac{\vec{b}+2\vec{d}}{3}$$

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

  Zero Marks : 0 If unanswered;

  Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks; choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2marks;

  choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get −2 marks.

[:Q.5] Let M = (a_{ij}), $M = (a_{ij}), i, j \in \{1,2,3\}$, be the 3×3 matrix such that $a_{ij} = 1$ if j + 1 is divisible by i, otherwise a_{ij} = 0. Then which of the following statements is(are) true ?

[:A] M is invertible

[:B] There exists a nonzero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that $M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$

[:C] The set $\{X \in \mathbb{R}^3 : MX = \mathbf{0}\} \neq \{\mathbf{0}\}$, where $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

[:D] The matrix (M –2I) is invertible, where I is the 3 × 3 identity matrix

[:ANS] BC

[:SOLN] $M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} : |M| = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix}$$

$\Rightarrow$ $a_1 + a_2 + a_3 = -a_1$; $a_1 + a_3 = -a_2$ and $a_2 = -a_3$

$\Rightarrow$ $2a_1 + a_2 + a_3 = 0$;

Determinant of Coefficients = $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0 \Rightarrow$ non-trivial solution.

mx = 0 $\Rightarrow$ $a_1 + a_2 + a_3 = 0$

$a_1 + a_3 = 0$; non- zero solution is present

$a_2 = 0$

$$|m - 2| = \begin{vmatrix} -1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 1 & -2 \end{vmatrix} = 0$$

[:Q.6] Let $f:(0,1)\to\mathbb{R}$ be the function defined as $f(x)=[4x]\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right)$, where [x] denotes the greatest integer less than or equal to x . Then which of the following statements is(are) true?

[:A] The function f is discontinuous exactly at one point in (0,1)

[:B] There is exactly one point in (0,1) at which the function f is continuous but **NOT** differentiable

[:C] The function f is **NOT** differentiable at more than three points in (0,1)

[:D] The minimum value of the function f is $-\frac{1}{512}$

[:ANS] AB

[:SOLN]

$$\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right)=x^3-x^2+\frac{5x}{16}-\frac{1}{32}$$

$$f(x)=\begin{cases}0 & 0<x<\frac{1}{4}\\ x^3-x^2+\frac{5x}{16}-\frac{1}{32}; & \frac{1}{4}\le x<\frac{1}{2}\\ 2\left(x^3-x^2+\frac{5x}{16}-\frac{1}{32}\right); & \frac{1}{2}\le x<\frac{3}{4}\\ 3\left(x^3-x^2+\frac{5x}{16}-\frac{1}{32}\right); & \frac{3}{4}\le x<1\end{cases}$$; not continuous at x = $\frac{3}{4}$

$$\Rightarrow f'(x)=\begin{cases}0 & ;0<x<\frac{1}{4}\\ 3x^2-2x+\frac{5}{16} & \frac{1}{4}\le x<\frac{1}{2}\\ 6x^2-4x+\frac{5}{8} & \frac{1}{2}\le x<\frac{3}{4}\\ 9x^2-6x+\frac{15}{16} & ;\frac{3}{4}\le x<1\end{cases}$$

$f'\left(\frac{1}{4}^+\right)=\frac{3}{16}-\frac{1}{2}+\frac{5}{16}=0\Rightarrow$ differentiable at x = $\frac{1}{4}$

$f'\left(\frac{1}{2}^-\right)=\frac{3}{4}-1+\frac{5}{16}=\frac{1}{16}$

$f'\left(\frac{1}{2}^+\right)=\frac{6}{4}-2+\frac{5}{8}=\frac{1}{8}\Rightarrow$ not differentiable at x = $\frac{1}{2}$

[:Q.7] Let S be the set of all twice differentiable functions f from R to R such that $\frac{d^2f}{dx^2}(x) > 0$ for all $x \in (-1, 1)$. For $f \in S$, let X_f be the number of points $x \in (-1,1)$ for which f(x) = x. Then which of the following statements is(are) true?

[:A] There exists a function $f \in S$ such that $X_f = 0$

[:B] For every function $f \in S$, we have $X_f \le 2$

[:C] There exists a function $f \in S$ such that $X_f = 2$

[:D] There does **NOT** exist any function f in S such that $X_f = 1$

[:ANS] ABC

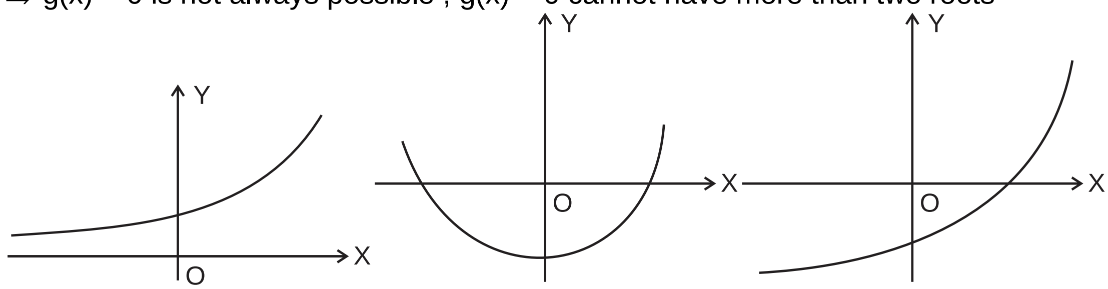
[:SOLN]

Let g(x) = f(x) – x

$g''(x) = f''(x) > 0$

The graph of y = g(x) is always concave upward

$\Rightarrow$ g(x) = 0 is not always possible ; g(x) = 0 cannot have more than two roots



SECTION 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If **ONLY** the correct integer is entered;

  Zero Marks : 0 In all other cases.

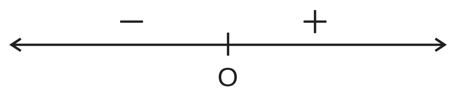
[:Q.8] For $x \in \mathbb{R}$, let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the minimum value of the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \int_0^{x\tan^{-1}x} \frac{e^{(t-\cos t)}}{1+t^{2023}} dt \text{ is}$$

[:ANS] 0

[:SOLN] $f'(x) = \frac{e^{x\tan^{-1}x} - \cos\left(x\tan^{-1}x\right)}{1+\left(x\tan^{-1}x\right)^{2023}}\left(\tan^{-1}x + \frac{x}{1+x^2}\right)$

sign scheme for f'(x)



minimum at x = 0

$\therefore$ f(0) = 0 ; Ans 0

[:Q.9] For $x \in R$, let y (x) be a solution of the differential equation $(x^2-5)\frac{dy}{dx} - 2xy = -2x(x^2-5)^2$ such that y(2) = 7.

Then the maximum value of the function y x ( ) is

[:ANS] 16

[:SOLN] $\frac{dy}{dx} + \left(\frac{-2x}{x^2-5}\right)y = -2x\left(x^2-5\right)$

I.F. = $e^{-\int\frac{2x}{x^2-5}dx} = e^{\ln\frac{1}{|x^2-5|}} = \frac{1}{|x^2-5|}$

Solutions is

$$\frac{y}{|x^2-5|} = -\int\frac{2x\left(x^2-5\right)}{|x^2-5|}dx$$

$$\Rightarrow \frac{y}{|x^2-5|} = -\int\frac{t}{|t|}dt \;; t = x^2-5$$

$$\Rightarrow \frac{y}{|x^2-5|} = -|x^2-5| + C$$

y(2) $\Rightarrow$ c – 1 = 7 $\Rightarrow$ c = 8

$\Rightarrow$ y = – $\left(|x^2-5|^2 - 8|x^2-5| + 16 - 16\right)$

$\Rightarrow$ y = 16 – $\left(|x^2-5| - 4\right)^2$

$\therefore$ $y_{min.}$ = 16

[:Q.10] Let X be the set of all five digit numbers formed using 1,2,2,2,4,4,0. For example, 22240 is in X while 02244 and 44422 are not in X . Suppose that each element of X has an equal chance of being chosen. Let p be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 5. Then the value of 38p is equal to

[:ANS] 31

[:SOLN] Sample Space : | | | | | 0 |

1, 2, 2, 2, 2, 4, 4

$$\left.\begin{array}{l} 1,2,2,2 \rightarrow 4 \\ 2,2,4,4 \rightarrow 6 \\ 1,2,2,4 \rightarrow 12 \\ 1,4,4,2 \rightarrow 12 \\ 2,2,2,4 \rightarrow 4 \end{array}\right\} 38$$

Favourable | | | | 2 | 0 | | | | | 4 | 0 |

$$\left.\begin{array}{l} 1,2,2,2,4 \\ 1,2,4 \rightarrow 6 \\ 2,2,2 \rightarrow 1 \\ 1,2,2 \rightarrow 3 \\ 2,2,4 \rightarrow 3 \end{array}\right\} 13$$

1, 2, 2, 4, 4

$$\left.\begin{array}{l} 1,2,4 \rightarrow 6 \\ 1,2,2 \rightarrow 3 \\ 1,4,4 \rightarrow 3 \\ 2,2,4 \rightarrow 3 \\ 2,4,4 \rightarrow 3 \end{array}\right\} 18$$

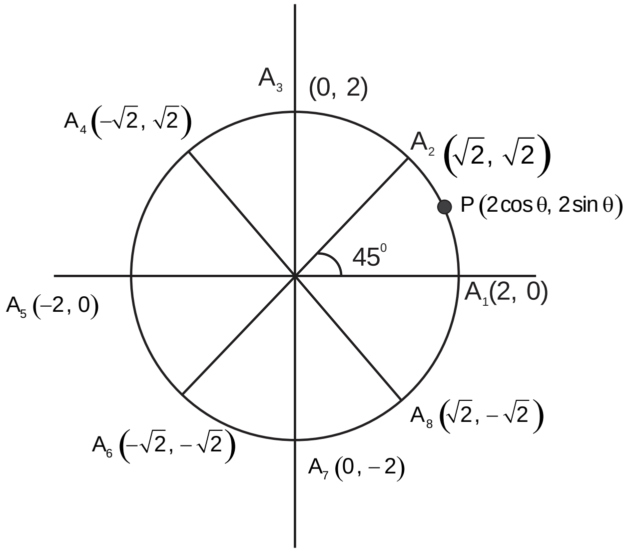
$$P = \frac{18+13}{38} = \frac{31}{38}$$

$\therefore$ 38P = 31

[:Q.11] Let $A_1, A_2, A_3, \ldots\ldots, A_8$ be the vertices of a regular octagon that lie on a circle of radius 2. Let P be a point on the circle and let PA_i denote the distance between the points P and A_i for i = 1,2, ……. ,8. If P varies over the circle, then the maximum value of the product $PA_1 \cdot PA_2 \ldots\ldots PA_8$, is

[:ANS] 512

[:SOLN]



$PA_2 \cdot PA_4 \cdot PA_6 \cdot PA_8 = \sqrt{\left(2\cos\theta \pm \sqrt{2}\right)^2 + \left(2\sin\theta \pm \sqrt{2}\right)^2}$ ....4 square roots

$= \sqrt{8 \pm 4\sqrt{2}\left(\cos\theta \pm \sin\theta\right)}$ . ...4 square roots

$= \sqrt{32 - 32\sin 2\theta} \cdot \sqrt{32 + 32\sin 2\theta} = 32\sqrt{1 - \sin^2 2\theta} = 32\cos 2\theta$

$PA_1PA_3PA_5PA_7 = \underbrace{\sqrt{\left(2\cos\theta \pm 2\right)^2 + 4\sin^2\theta}}_{\text{2 square roots}} \cdot \underbrace{\sqrt{\left(2\sin\theta \pm 2\right)^2 + 4\cos^2\theta}}_{\text{2 square roots}}$

$= \sqrt{8 \pm 8\cos\theta} \cdot \sqrt{8 \pm 8\sin\theta}$

$= 8\sqrt{\sin^2\theta} \cdot 8\sqrt{\cos^2\theta} = 32\sin 2\theta$

required product = 32.16-2 sin2θcos2θ = 512 sin4θ

$\therefore$ Required Maximum value = 512

[:Q.12] Let $R = \left\{ \begin{pmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{pmatrix} : a, b, c, d \in \{0,3,5,7,11,13,17,19\} \right\}$. Then the number of invertible matrices in R is

[:ANS] 3780

[:SOLN] Total no of matrices = 8^4

Det = 0 $\quad$ 0.3 = 0.3 $\rightarrow$ 7 × 7 × 2 × 2 = 196

0.3 = 0.0 $\rightarrow {}^7C_2 \times 2 \times 2 = 28$;

3.5 = 3.5 $\rightarrow {}^7C_2 \times 2 \times 2 = 84$

A = b = c = d → 8

∴ No. of invertible matrices = $8^4 - 196 - 28 - 84 - 8 = 3780$

[:Q.13] Let C_1 be the circle of radius 1 with center at the origin. Let C_2 be the circle of radius r with center at the point A = (4,1), where 1 < r < 3. Two distinct common tangents PQ and ST of C_1 and C_2 are drawn. The tangent PQ touches C_1 at P and C_2 at Q. The tangent ST touches C_1 at S and C_2 at T. Mid points of the line segments PQ and ST are joined to form a line which meets the x -axis at a point B . If $AB = \sqrt{5}$, then the value of r^2 is

[:ANS] 2

[:SOLN] $x^2 + y^2 - 1 = 0$; $x^2 + y^2 - 8x - 2y + 17 - r^2 = 0$

AB is radical axis :

Equation of AB is

$-1 + 8x + 2y - 17 + r^2 = 0$

$\Rightarrow 8x + 2y + r^2 - 18 = 0$

$\therefore \quad B \equiv \left(\frac{18 - r^2}{8}, 0\right) \quad ; \quad A \equiv (4,1)$

$AB^2 = 5 \Rightarrow \left(\frac{18 - r^2}{8} - 4\right)^2 + 1 = 5$

$\Rightarrow \left(\frac{18 - r^2}{8} - 4\right)^2 = 4 \Rightarrow \left(\frac{r^2 + 14}{8}\right)^2 = 4$

$\Rightarrow r^2 = 2$

SECTION 4 (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;

  Zero Marks : 0 In all other cases.

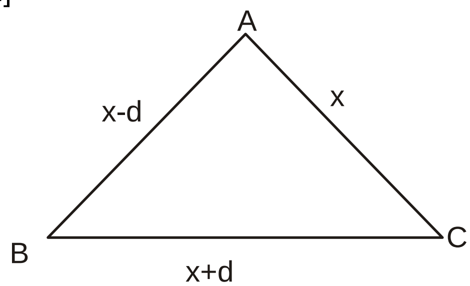
Passage-1

Consider an obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\pi/2$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1

[:Q.14] Let a be the area of the triangle ABC. Then the value of $(64a)^2$ is

[:ANS] 1008

[:SOLN]



$$A - C = \frac{\pi}{2}$$

$$A = \frac{\pi}{2} + C$$

$$B = \pi - (A + C) = \pi - \left(\frac{\pi}{2} + 2C\right) = \frac{\pi}{2} - 2C$$

2AC = AB+ BC

4RsinB= 2RsinC + 2RsinA

2cos2C = sinC + cosC

$$\cos C - \sin C = \frac{1}{2} \Rightarrow \sin 2C = \frac{3}{4}$$

$$\text{Now, } a = \frac{AB.BC.CA}{4R} = 2R^2 \sin A \sin B \sin C = \sin 2C \cos 2C$$

$$a = \frac{3}{4}\frac{\sqrt{7}}{4}$$

$$\therefore 64a = 12\sqrt{7} \Rightarrow (64a)^2 = 1008$$

Passage-1

Consider an obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\pi/2$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1

[:Q.15] Then the inradius of the triangle ABC is

[:ANS] 0.25

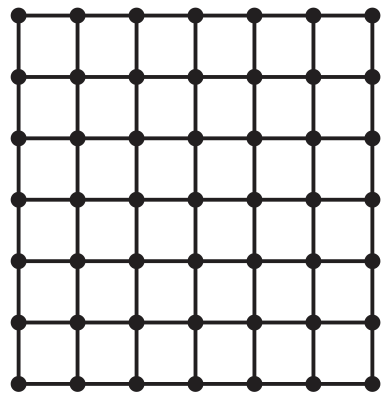
[:SOLN] $r = \frac{\Delta}{s} = \frac{a}{AB + BC + CA} = \frac{\sin 2C \ \text{Cos}2C}{2R\left(\text{SinA} + \text{SinB} + \text{SInC}\right)}$

$= = \frac{\sin 2C \ \text{Cos}2C}{2\left(\text{CosC} + \text{Cos}2C + \text{SinC}\right)}$

$= \frac{\text{Sin}2C \cdot \text{Cos}2C}{2 \cdot 3\text{Cos}2C} = \frac{\sin 2C}{6} = \frac{1}{8}$

Passage-2

Consider the 6 × 6 square in the figure. Let A_1, A_2, ……, A_{49}, be the points of intersections (dots in the picture) in some order. We say that A_i and A_j are friends if they are adjacent along a row or along a column. Assume that each point A_i has an equal chance of being chosen.



[:Q.16] Let p_i be the probability that a randomly chosen point has i many friends, i = 0,1,2,3,4. Let X be a random variable such that for i = 0,1,2,3,4 , the probability P(X = i) = p_i . Then the value of 7E(X) is

[:ANS] 24

[:SOLN] P(0) = 0 P(1) = 0 P(2) = $\frac{4}{49}$ P(3) = $\frac{20}{49}$

P(4) = 1 – $\frac{24}{49} = \frac{25}{49}$

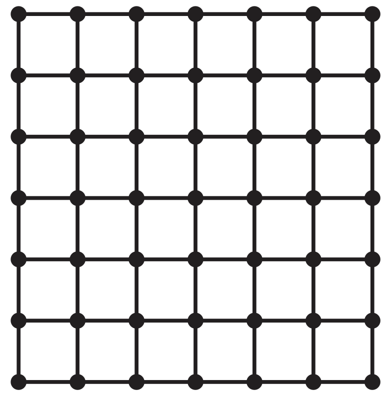
E(X) = $\sum_{i=0}^{4} i \cdot P(i) = 0.0 + 1.0 + 2 \cdot \frac{4}{49} + 3 \cdot \frac{20}{49} + 4 \cdot \frac{25}{49}$

$= \frac{8 + 60 + 100}{49} = \frac{168}{49} = \frac{24}{7}$

7E(x) = $7 \times \frac{24}{7} = 24$

Passage-2

Consider the 6 × 6 square in the figure. Let A_1, A_2, ……, A_{49}, be the points of intersections (dots in the picture) in some order. We say that A_i and A_j are friends if they are adjacent along a row or along a column. Assume that each point A_i has an equal chance of being chosen.



[:Q.17] Two distinct points are chosen randomly out of the points A_1, A_2, ….., A_{49}. Let p be the probability that they are friends. Then the value of 7 p is

[:ANS] 0.50

[:SOLN] $$p = \frac{(6\times7)+(6\times7)}{^{49}C_2} = \frac{2\times(6\times7)\times2}{49\times48} = \frac{1}{14}$$

$$7p = 7 \times \frac{1}{14} = \frac{7}{14} = \frac{1}{2} = 0.5$$