JEE (ADVANCED) 2023 PAPER-1

[PAPER WITH SOLUTION]

HELD ON SUNDAY 04TH JUNE 2023

PHYSICS

SECTION 1 (Maximum Marks : 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 mark;

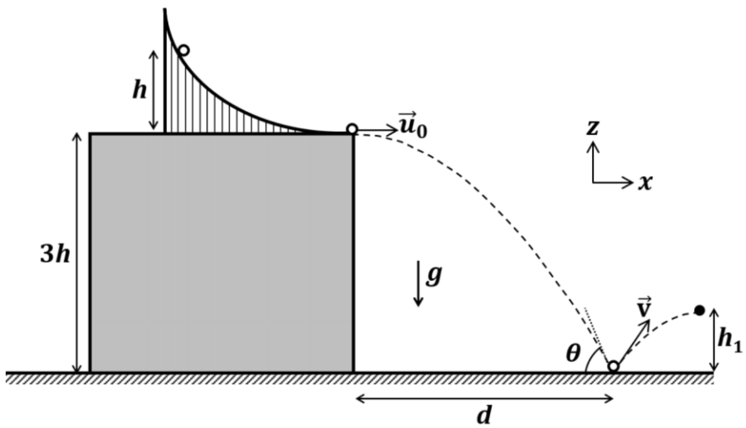
choosing ONLY (B) will get +1 mark;

choosing ONLY (D) will get +1 mark;

choosing no option (i.e. the question is unanswered) will get 0 marks; and

choosing any other combination of options will get −2 marks.

[:Q.1] A slide with a frictionless curved surface, which becomes horizontal at its lower end, is fixed on the terrace of a building of height $3h$ from the ground, as shown in the figure. A spherical ball of mass is released on the slide from rest at a height h from the top of the terrace. The ball leaves the slide with a velocity $\vec{u}_0 = u_0 x$ and falls on the ground at a distance from the building making an angle with the horizontal. It bounces off with a velocity $\vec{v}$ and reaches a maximum height h_1. The acceleration due to gravity is g and the coefficient of restitution of the ground is $\frac{1}{\sqrt{3}}$. Which of the following statement(s) is(are) correct?



[:A] $\vec{u}_0 = \sqrt{2gh}\, x$

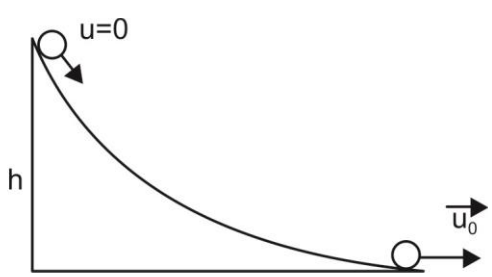
[:B] $\vec{v} = \sqrt{2gh}\left(x - z\right)$

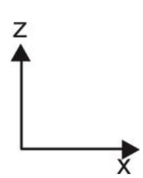
[:C] $\theta = 60^\circ$

[:D] $d / h_1 = 2\sqrt{3}$

[:ANS] ACD

[:SOLN]



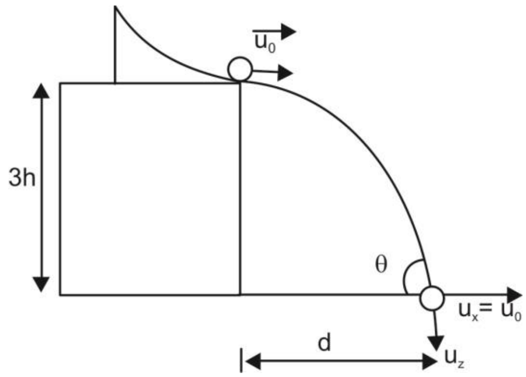


On the frictionless curved surface

$$mgh = \frac{1}{2} m u_0^2$$

$u_0 = \sqrt{2gh}$

$\vec{u}_0 = \sqrt{2gh}x$



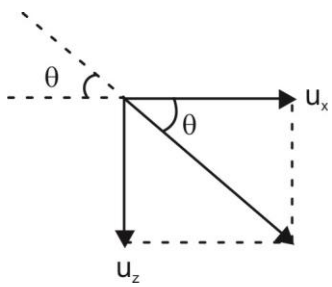
after leaving the frictionless curved surface

motion in z –direction

$v^2 = u^2 + 2gH$

$u_z^2 = 0 + 2g3h$

$u_z = \sqrt{6gh}$



$\tan\theta = \frac{u_z}{u_x} = \frac{\sqrt{6gh}}{\sqrt{2gh}} = \sqrt{3}$

$\theta = 60°$

Horizontal displacement 'd' during projectile motion

$x = u_x t$

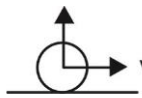
$d = u_0\sqrt{\frac{2(3h)}{g}}$

$d = \sqrt{2gh}\sqrt{\frac{6R}{g}}$

$d = 2\sqrt{3}\,h$

after collision

$v_z = eu_z$



$v_z = eu_z$

$= \frac{1}{\sqrt{3}}\sqrt{6gh}$

$v_z = \sqrt{2gh}$

max height attain after collision

$h_1 = \frac{v_z^2}{2g} = \frac{2gh}{2g}$

$h_1 = h$

so

$\frac{d}{h_1} = \frac{2\sqrt{3}h}{h}$

$\frac{d}{h_1} = 2\sqrt{3}$

and after collision

$\vec{v} = v_x x + v_z z = \sqrt{2gh}x + \sqrt{2gh}z$

$\vec{v} = \sqrt{2gh}(x + z)$

[:Q.2] A plane polarized blue light ray is incident on a prism such that there is no reflection from the surface of the prism. The angle of deviation of the emergent ray is $\delta = 60°$ (see Figure-1). The angle of minimum deviation for red light from the same prism is $\delta_{min} = 30°$ (see Figure-2). The refractive index of the prism material for blue light is $\sqrt{3}$. Which of the following statement(s) is(are) correct?

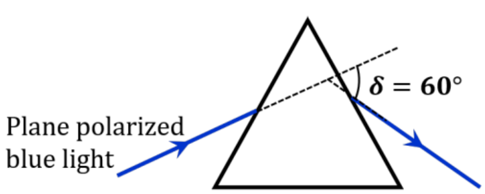


Figure-1

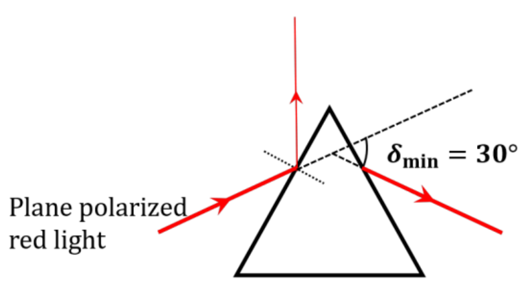


Figure-2

[:A] The blue light is polarized in the plane of incidence.

[:B] The angle of the prism is 45°.

[:C] The refractive index of the material of the prism for red light is $\sqrt{2}$

[:D] The angle of refraction for blue light in air at the exit plane of the prism is 60°.

[:ANS] **ACD**

[:SOLN] For blue polarized light there are no reflection i.e. blue polarized light incident at Brewster angle.

$\mu = \tan i_p$

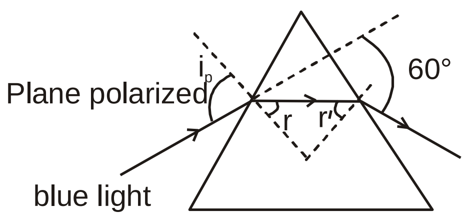
$\sqrt{3} = \tan i_p$

$i_p = 60°$

Now angle of deviation $\delta = i + e - A$

$60° = 60° + e - A$

$e = A ....(i)$



Snell's law at incident surface

$1 \sin i_p = \sqrt{3} \sin r$

$1 \sin 60° = \sqrt{3} \sin r$

$\sin r = \frac{1}{2}$

$r = 30°$

since $r + r' = A$

$r' = A - r$

$r' = A - 30°$

Snell's law at emergent surface

$\sqrt{3} \sin r' = 1 \sin e$

$\sqrt{3} \sin(A - 30°) = \sin A$

since e = A from equation (i)

Now $\sqrt{3}\sin(A-30°)=\sin A$

$$\sqrt{3}\{\sin A\cos 30° - \cos A\sin 30°\} = \sin A$$

$$\sqrt{3}\left\{\sin A\frac{\sqrt{3}}{2} - \cos A\frac{1}{2}\right\} = \sin A$$

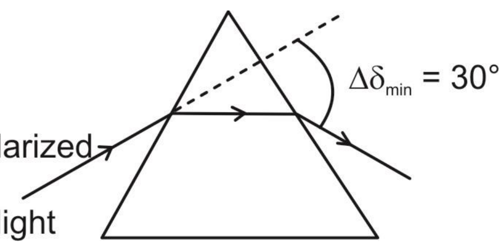
$$\frac{3}{2}\sin A - (\cos A)\frac{\sqrt{3}}{2} = \sin A$$

$$\frac{1}{2}\sin A = \frac{\sqrt{3}}{2}\cos A$$

$$\tan A = \sqrt{3}$$

$$A = 60°$$

from equation (i) A = e= 60°



For plane polarized red light.

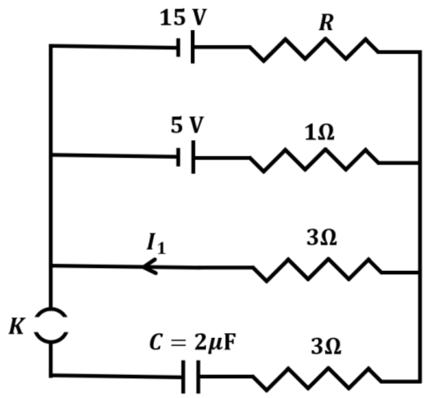
$$\mu_R = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\mu_R = \frac{\sin\left(\frac{60°+30}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$\mu_R = \sqrt{2}$$

[:Q.3] In a circuit shown in the figure, the capacitor is initially uncharged and the key is open. In this condition, a current of 1 A flows through the 1 Ω resistor. The key is closed at time $t = t_0$. Which of the following statement(s) is(are) correct?

[Given: $e^{-1} = 0.36$]



[:A] The value of the resistance R is 3 Ω.

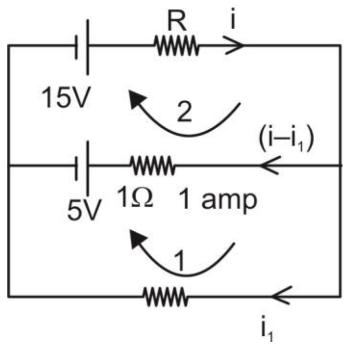
[:B] For $t < t_0$, the value of current I_1 is 2 A.

[:C] At $t = t_0 + 7.2\mu s$, the current in the capacitor is 0.6 A.

[:D] For $t \to \infty$, the charge on the capacitor is 12 μ.

[:ANS] ABCD

[:SOLN] Before key closed (t < t_0)



KVL in loop (i)

$\sum v = 0$

$3i_1 - 5 - 1 \times 1 = 0$

$i_1 = 2$ Amp

since $i - i_1 = 1$

$i - 2 = 1 \Rightarrow i = 3$ Amp

Now KVL in loop (ii)

$\sum v = 0$

$-15 + iR + 1\times 1 + 5 = 0$

$-9 + 3R = 0$

$R = 3\Omega$

Equivalent emf $E_{eq} = \dfrac{\sum \frac{E}{r}}{\sum \frac{1}{r}}$

$$E_{eq} = \frac{\frac{15}{2} + \frac{5}{1} + \frac{0}{3}}{\frac{1}{R} + \frac{1}{1} + \frac{1}{3}} = 6V$$

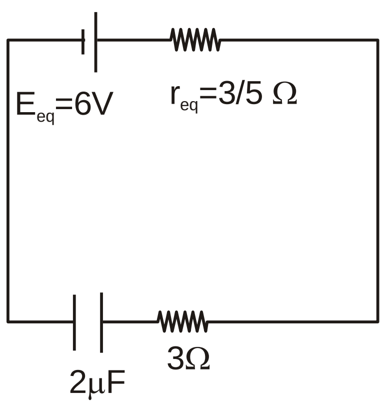
Equivalent Resistance

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{1} + \frac{1}{3} = \frac{1}{3} + 1 + \frac{1}{3}$$

.

$$R_{eq} = \frac{3}{5}\Omega$$

After key closed $t \geq t_0$



Steady state charge $Q_0 = CE_{eq} = 2\times 6 = 12\mu C$

Time constant for charging of capacitor

$\tau = R_{eq}C$

$$\tau = \left(\frac{3}{5} + 3\right)\times 2\times 10^{-6} = 7.2\times 10^{-6}\text{ sec}$$

During charging of capacitor instantaneous charge stored in capacitor $q = Q_0\left(1 - e^{-t/\tau}\right)$

instantaneous current $i = \dfrac{dq}{dt}$

$$i = \frac{Q_0}{\tau}e^{-t/\tau}$$

at $t = t_0 + 7.2\mu s$

$$i = \frac{Q_0}{\tau} e^{-1} = \frac{12 \times 10^{-6}}{7.2 \times 10^{-6}} \times 0.36 = 0.6\,\text{Amp}$$

SECTION2 (Maximum Marks :12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

[:Q.4] A bar of mass $M = 1.00\text{kg}$ and length $L = 0.20\,\text{m}$ is lying on a horizontal frictionless surface. One end of the bar is pivoted at a point about which it is free to rotate. A small mass $m = 0.10\,\text{kg}$ is moving on the same horizontal surface with $5.00\,\text{ms}^{-1}$ speed on a path perpendicular to the bar. It hits the bar at a distance $L/2$ from the pivoted end and returns back on the same path with speed v. After this elastic collision, the bar rotates with an angular velocity ω. Which of the following statement is correct?

[:A] $\omega = 6.98$ rad s^{-1} and $v = 4.30\,ms^{-1}$

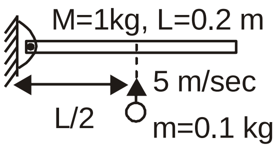
[:B] $\omega = 3.75$ rad s^{-1} and $v = 4.30\,ms^{-1}$

[:C] $\omega = 3.75$ rad s^{-1} and $v = 10.0\,ms^{-1}$

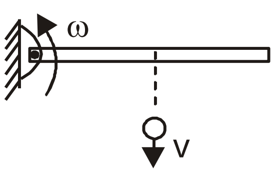
[:D] $\omega = 6.80$ rad s^{-1} and $v = 4.10\,ms^{-1}$

[:ANS] **A**

[:SOLN] **Before collision**



After collision



Conservation of angular momentum about point of pivot

$\vec{L}_i = \vec{L}_f$

$$mu\frac{L}{2} = -mv\frac{L}{2} + I\omega$$

$$0.1\times 5\times \frac{0.2}{2} = -0.1V\times \frac{0.2}{2} + \frac{1(0.2)^2}{3}\omega$$

$$\frac{0.2}{3}\omega - 0.05v = 0.25 \ldots\ldots(i)$$

coefficient of restitution

$$e = 1 = \frac{\frac{L}{2}\omega + v}{u - 0}$$

$$1 = \frac{\frac{0.2}{2}\omega + v}{5}$$

$$v + 0.1\omega = 5 \ldots.(ii)$$

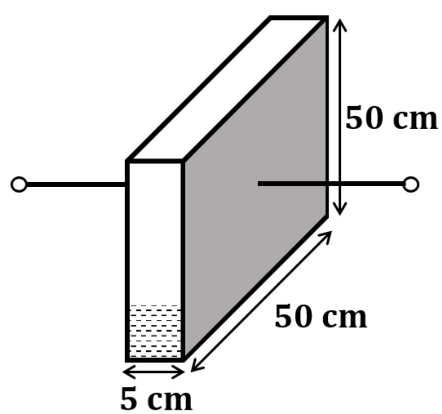
solving equation (i) and (ii)

v = 4.30 m/sec

$\omega = 6.98$ rad / sec

[:Q.5] A container has a base of 50 cm × 5 cm and height 50 cm, as shown in the figure. It has two parallel electrically conducting walls each of area 50 cm × 50 cm. The remaining walls of the container are thin and non-conducting. The container is being filled with a liquid of dielectric constant 3 at a uniform rate of 250 cm^3 s^{-1}. What is the value of the capacitance of the container after 10 seconds?

[Given: Permittivity of free space $\epsilon_0 = 9 \times 10^{-12}$ C $2N^{-1}$ m^{-2}, the effects of the non-conducting walls on the capacitance are negligible]



[:A] 27 pF

[:B] 63 pF

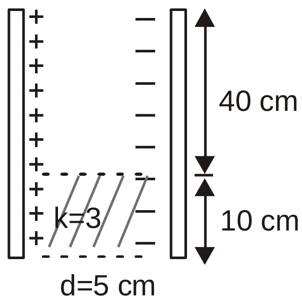
[:C] 81 pF

[:D] 135 pF

[:ANS] **B**

[:SOLN] In 10 sec volume of liquid filled = 250×10 = 2500 cm^3

Height of liquid raised h = $\frac{2500}{5\times50}$ = 10 cm



Equivalent capacitance

$C = C_1 + C_2$

$$C = \frac{A_1 \in_0 K}{d} + \frac{A_2 \in_0}{d} = \frac{\left(50\times10\times10^{-4}\right)\in_0 \times3}{5\times10^{-2}} + \frac{50\times40\times10^{-4}}{5\times10^{-2}}\in_0$$

$$= 7\times9\times10^{-12} = 63\times10^{-12}F = 63PF$$

[:Q.6] One mole of an ideal gas expands adiabatically from an initial state (T_A, V_0) to final state $(T_f, 5V_0)$. Another mole of the same gas expands isothermally from a different initial state (T_B, V_0) to the same final state $(T_f, 5V_0)$. The ratio of the specific heats at constant pressure and constant volume of this ideal gas is γ. What is the ratio T_A / T_B?

[:A] $5^{\gamma-1}$

[:B] $5^{1-\gamma}$

[:C] 5^{γ}

[:D] $5^{1+\gamma}$

[:ANS] **A**

[:SOLN] $\because$ In adiabatic expansion

$TV^{\gamma-1} = \text{constant}$

$\therefore T_A V_0^{\gamma-1} = (T_f)(5V_0)^{\gamma-1}$

$T_A = (T_f)(5)^{\gamma-1}$ ....(i)

$\because$ In isothermal process

Temperature = constant

$\therefore$ $T_B = T_f$ ....(ii)

From (i) and (ii)

$\Rightarrow T_A = T_B(5)^{\gamma-1}$

$\therefore T_A / T_B = (5)^{\gamma-1}$

[:Q.7] Two satellites P and Q are moving in different circular orbits around the Earth (radius R. The heights of P and Q from the Earth surface are h_P and h_Q, respectively, where $h_p = R/3$. The accelerations of P and Q due to Earth's gravity are g_p and g_Q, respectively. If $g_p / g_Q = 36/25$, what is the value of h_Q ?

[:A] $3R/5$

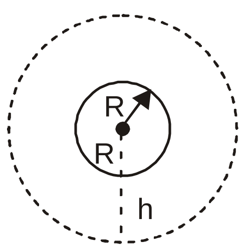
[:B] $R/6$

[:C] $6R/5$

[:D] $5R/6$

[:ANS] **A**

[:SOLN]



$\therefore$ Acceleration due to gravity $(g) = \dfrac{GM}{(R+h)^2}$

$$\therefore \frac{g_p}{g_Q} = \left(\frac{R+h_Q}{R+h_p}\right)^2$$

$$\Rightarrow \sqrt{\frac{36}{25}} = \left(\frac{R+h_Q}{R+R/3}\right) \Rightarrow \frac{6}{5} = \frac{R+h_Q}{\frac{4R}{3}} \qquad \Rightarrow \frac{6}{5}\left(\frac{4R}{3}\right) - R = h_Q$$

$$\therefore h_Q = \frac{3R}{5}$$

SECTION3 (Maximum Marks :24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If ONLY the correct integer is entered;

  Zero Marks : 0 In all other cases.

[:Q.8] A Hydrogen-like atom has atomic number Z Photons emitted in the electronic transitions from level $n = 4$ to level $n = 3$ in these atoms are used to perform photoelectric effect experiment on a target metal. The maximum kinetic energy of the photoelectrons generated is 1 . 95 eV. If the photoelectric threshold wavelength for the target metal is 310 nm, the value of is _______

[Given: $hc = 1240$ eV-nm and $Rhc = 13.6$ eV, where is the Rydberg constant, h is the Planck's constant and is the speed of light in vacuum]

[:ANS] 3

[:SOLN] Energy of photon emitted is $\Delta E = (13.6\text{eV})(z^2)\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = (13.6\text{eV})(z^2)\left(\frac{7}{9 \times 16}\right)$.........(i)

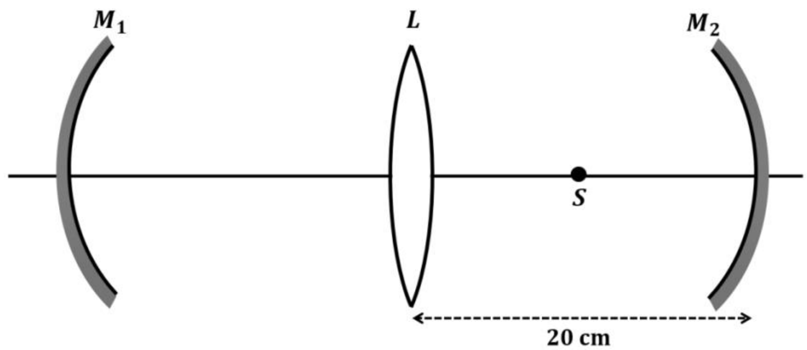
$\because$ Work function $(\phi) = \frac{hc}{\lambda_{th}} = \frac{1240\text{eV nm}}{310\text{nm}} = 4\text{eV}$

$\because \Delta E = \phi + (\text{K.E.})_{max}$

$13.6(z^2)\left(\frac{7}{9 \times 16}\right) = 4 + 1.95$

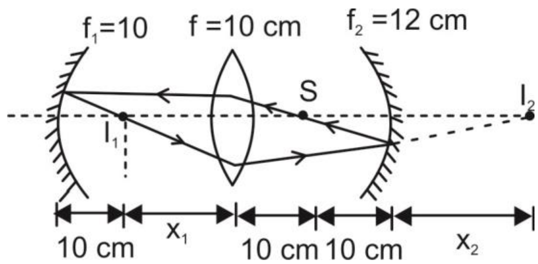
$\therefore z = 3$

[:Q.9] An optical arrangement consists of two concave mirrors M_1 and M_2, and a convex lens L with a common principal axis, as shown in the figure. The focal length of L is 10 cm. The radii of curvature of M_1 and M_2 are 20 cm and 24 cm, respectively. The distance between L and M_2 is 20 cm. A point object S is placed at the mid-point between L and M_2 on the axis. When the distance between L and M_1 is $n/7$ cm, one of the images coincides with S. The value of n is _______.



[:ANS] 80 or 150 or 220

[:SOLN] Possibility 1



for mirror M_2

$$u = x_2, v = -10\text{cm}, f = -12\text{cm}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-10} + \frac{1}{x_2} = \frac{1}{-12}$$

$$\Rightarrow \frac{1}{x_2} = \frac{1}{10} - \frac{1}{12} = \frac{12-10}{120} = \frac{2}{120} = \frac{1}{60}$$

$$\therefore x_2 = 60\,\text{cm}$$

for lens

$$u = -x, v = (20\,\text{cm} + x_2) = 80\,\text{cm}, \text{ and } f = +10\,\text{cm}$$

$$\because \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{80} - \frac{1}{-x_1} = \frac{1}{10}$$

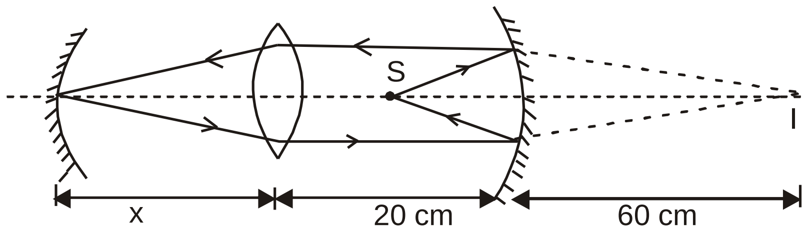
$$\Rightarrow \frac{1}{x_1} = \frac{1}{10} - \frac{1}{80} = \frac{8-1}{80} = \frac{7}{80}$$

$$\therefore x_1 = \frac{80}{7}\text{cm}$$

$$\therefore \text{ distance = 10 cm } + x_1 = 10\,\text{cm} + \frac{80}{7}\text{cm} = \frac{150}{7}\text{cm}$$

$\therefore n = 150$

Possibility (2)



for lens

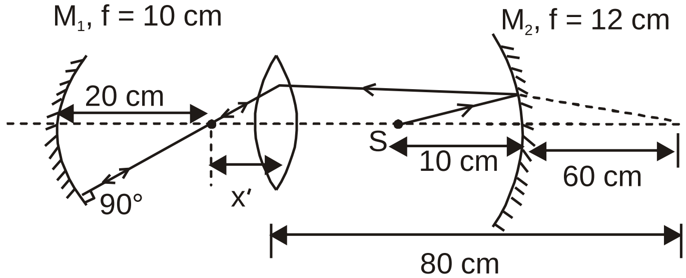
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{x} - \frac{1}{-80} = \frac{1}{10}$$

$$\frac{1}{x} = \frac{1}{10} - \frac{1}{80} = \frac{8-1}{80} = \frac{7}{80}$$

$$x = \frac{80}{7} \text{cm}$$

Ans.80

Possibility -3



for M_2

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{-10} = \frac{1}{-12}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{12} = \frac{12-10}{120} = \frac{2}{120} = \frac{1}{60}$$

$v = 60 \text{cm}$

for lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{x} - \frac{1}{-80} = \frac{1}{10} \Rightarrow \frac{1}{x} = \frac{1}{10} - \frac{1}{80}$$

$$\Rightarrow \frac{1}{x} = \frac{8-1}{80} = \frac{7}{80} \Rightarrow k = \frac{80}{7}\text{cm}$$

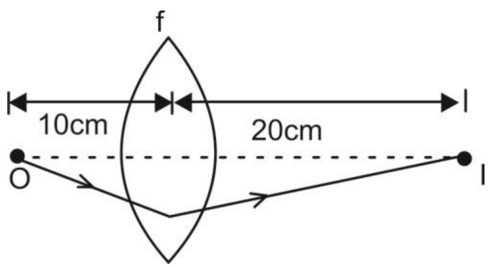
$$\therefore \text{distance} = 20\text{ cm} + \frac{80}{7}\text{cm} = \frac{140\text{cm}+80\text{cm}}{7} = \frac{220}{7}\text{cm}$$

ans. 220

[:Q.10] In an experiment for determination of the focal length of a thin convex lens, the distance of the object from the lens is $10 \pm 0.1\text{cm}$ and the distance of its real image from the lens I $20 \pm 0.2\text{cm}$ cm. The error in the determination of focal length of the lens is n %. The value of n is _______.

[:ANS] 1

[:SOLN]



$$\because \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{20} - \frac{1}{-10} = \frac{1}{f}$$

$$\frac{1+2}{20} = \frac{1}{f} \Rightarrow f = \frac{20}{3}\text{cm}$$

$$\because -\frac{1}{v^2}dv + \frac{1}{u^2}du = -\frac{1}{f^2}df$$

$$\therefore \frac{\Delta f}{f^2} = \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}$$

$$\Rightarrow \frac{\Delta f}{f} \times 100 = f \times 100\left(\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}\right)$$

$$= \frac{20}{3} \times 100\left(\frac{0.1}{(10)^2} + \frac{(0.2)}{(20)^2}\right) = \frac{20}{3}\left(0.1 + \frac{0.2}{7}\right) = \frac{20}{3}\left(\frac{0.6}{4}\right) = 1\%$$

[:Q.11] A closed container contains a homogeneous mixture of two moles of an ideal monatomic gas $(\gamma = 5/3)$ and one mole of an ideal diatomic gas $(\gamma = 7/5)$. Here, γ is the ratio of the specific heats at constant pressure and constant volume of an ideal gas. The gas mixture does a work of 66 Joule when heated at constant pressure. The change in its internal energy is ________ Joule.

[:ANS] 121

[:SOLN] 2 moles of ideal monoatomic gas

$$\gamma = \frac{5}{3}$$

1 mole of an ideal diatomic gas $\left(\gamma = \frac{7}{5}\right)$

Gasmix does work of 60 joule at const P

$\Delta U = ?$

$$\gamma_{mix} = \frac{n_1 CP_1 + n_2 CP_2}{n_1 CV_1 + n_2 CV_2} = \frac{2\times\frac{5}{2}R + 1\times\frac{7}{2}R}{2\times\frac{3}{2}R + 1\times\frac{5}{2}R} = \frac{17}{11}$$

$$\left.\begin{array}{l} W = nR\Delta T \\ \Delta u = nC_v\Delta T \\ Q : nC_p dT \end{array}\right\} Q = \Delta U + W$$

$$\frac{\Delta u}{W} = \frac{\Delta u}{\theta - \Delta U}$$

$$\frac{\Delta u}{W} = \frac{1}{\frac{Q}{\Delta u} - 1}$$

$$\frac{\Delta u}{W} = \frac{1}{\frac{C_P}{C_V} - 1}$$

$$\frac{\Delta u}{W} = \frac{1}{\frac{17}{11} - 1}$$

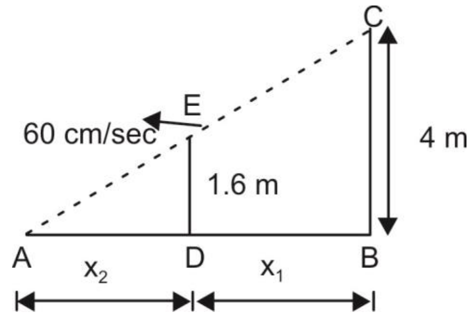
$$\frac{\Delta u}{66} = \frac{11}{6}$$

$$\Delta u = 121 \text{Joule}$$

[:Q.12] A person of height 1.6 m is walking away from a lamp post of height 4 m along a straight path on the flat ground. The lamp post and the person are always perpendicular to the ground. If the speed of the person is 60 cm s^{-1}, the speed of the tip of the person's shadow on the ground with respect to the person is _______ cm s^{-1}.

[:ANS] 40

[:SOLN]



By similarity of ΔABC and ΔADE

$$\frac{x_1 + x_2}{x_2} = \frac{4}{1.6}$$

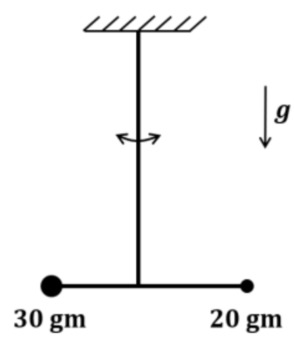
$$\frac{x_1 + x_2}{x_2} = \frac{5}{2}$$

$$x_2 = \frac{2}{3} x_1$$

$$\frac{dx_2}{dt} = \frac{2}{3}\frac{dx_1}{dt}$$

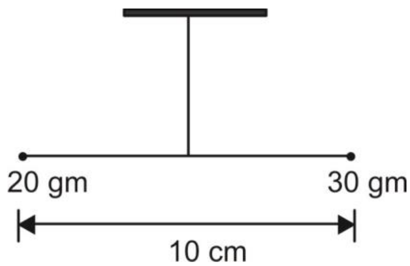
$$\frac{dx_2}{dt} = \frac{2}{3} \times 60 = 40\,\text{cm/sec}$$

[:Q.13] Two point-like objects of masses 20 gm and 30 gm are fixed at the two ends of a rigid massless rod of length 10 cm. This system is suspended vertically from a rigid ceiling using a thin wire attached to its center of mass, as shown in the figure. The resulting torsional pendulum undergoes small oscillations. The torsional constant of the wire is $1.2 \times 10^{-8}\,\text{N m rad}^{-1}$. The angular frequency of the oscillations in $n \times 10^{-3}\,\text{rad s}^{-1}$. The value of n is _____.



[:ANS] 10

[:SOLN]



Given : $c = 1.2 \times 10^{-8}$

$$M_{eq} = \frac{M_1 M_2}{M_1 + M_2} = \frac{20 \times 30}{20 + 30} = 12\text{gm} = 12 \times 10.3\text{kg}$$

$$I_{cm} = M_{eq} r^2 = 12 \times 10^{-3} \times (0.1)^2 = 12 \times 10^{-5}\text{kg} - \text{m}^2$$

$$T = 2\pi \sqrt{\frac{I_{cm}}{C}}$$

$$\frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{I_{cm}}{C}}$$

$$\omega_n = \sqrt{\frac{C}{I_m}}$$

$$\omega_n = \sqrt{\frac{1.2 \times 10^{-8}}{12 \times 10^{-5}}} = 10 \times 10^{-3}\text{rad/sec} = n \times 10^{-3}\text{rad/sec}$$

$$\therefore n = 10$$

SECTION4 (Maximum Marks :12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases

[:Q.14] List-I shows different radioactive decay processes and List-II provides possible emitted particles. Match each entry in List-I with an appropriate entry from List-II, and choose the correct option.

LIST-I		LIST-II
(P) $^{238}_{92}U \rightarrow ^{234}_{91}Pa$	(1)	one α particle and one β^+ particle
(Q) $^{214}_{82}Pb \rightarrow ^{210}_{82}Pb$	(2)	three β^- particles and one α particle
(R) $^{210}_{81}Tl \rightarrow ^{206}_{82}Pb$	(3)	two β^- particles and one α particle
(S) $^{228}_{91}Pa \rightarrow ^{224}_{88}Ra$	(4)	one α particle and one β^- particle
	(5)	one α particle and two β^+ particles

Codes :

[:A] $P \rightarrow 4, Q \rightarrow 3, R \rightarrow 2, S \rightarrow 1$

[:B] $P \rightarrow 4, Q \rightarrow 1, R \rightarrow 2, S \rightarrow 5$

[:C] $P \rightarrow 5, Q \rightarrow 3, R \rightarrow 1, S \rightarrow 4$

[:D] $P \rightarrow 5, Q \rightarrow 1, R \rightarrow 3, S \rightarrow 2$

[:ANS] A

[:SOLN]

(P) $U^{238}_{92} \rightarrow Pa^{234}_{91}$

Mass no diff: 238 – 234 = 4

α decay responsible so (1 α decay)

Atomic no diff to 1α decay (92 – 2) = 90

$X^{234}_{90} \xrightarrow{\beta^-} Pa^{234}_{91}$

β^- decay responsible for 1 increment in atomic no.

$\therefore$ $1\alpha, 1\beta^-$ decay P -4

(Q) $Pb^{214}_{82} \longrightarrow Pb^{210}_{82}$

Mass no (214 – 210) = 4, 1α decay

$X^{210}_{80} \xrightarrow{2\beta^{-1}} Pb^{210}_{82}$

$\therefore$ $1\alpha, 2\beta^-$ decay Q-3

(R) $Tl^{210}_{81} \longrightarrow Pb^{206}_{82}$

Mass No. (210 – 206) = 4

$\therefore 1\alpha$ decay

$X_{79}^{206} \xrightarrow{3\beta^{-1}} Pb_{82}^{206}$

$\therefore 1\alpha, 3\beta^{-}$ decay R-2

(S) $Pa_{91}^{228} \longrightarrow Ra_{88}^{224}$

Mass No. (228 -224) = 4

$1\,\alpha$ decay

$\therefore X_{89}^{224} \xrightarrow{\beta^{+}} Ra_{88}^{224}$

$1\alpha, 1\beta^{+}$ decay S-1

[:Q.15] Match the temperature of a black body given in List-I with an appropriate statement in List-II, and choose the correct option.

[Given: Wien's constant as $2.9\times10^{-3}\,m-K$ and $\frac{hc}{e}=1.24\times10^{-6}\,V-m$]

LIST-I	LIST-II
(P) 2000 K	(1) The radiation at peak wavelength can lead to emission of photoelectrons from a metal of work function 4 eV.
(Q) 3000 K	(2) The radiation at peak wavelength is visible to human eye.
(R) 5000 K	(3) The radiation at peak emission wavelength will result in the widest central maximum of a single slit diffraction.
(S) 10000 K	(4) The power emitted per unit area is 1/16 of that emitted by a blackbody at temperature 6000 K.
	(5) The radiation at peak emission wavelength can be used to image human bones.

[:A] $P\to3,\ Q\to5,\ R\to2,\ S\to3$

[:B] $P\to3,\ Q\to2,\ R\to4,\ S\to1$

[:C] $P\to3,\ Q\to4,\ R\to2,\ S\to1$

[:D] $P\to1,\ Q\to2,\ R\to5,\ S\to3$

[:ANS] C

[:SOLN] Given: $\frac{hc}{e} = 1.24 \times 10^{-6} V - m$

$b = 2.9 \times 10^{-3} m - K$

By weins law

$\lambda.T = b$

$\lambda = \frac{b}{T}$

and $E = \frac{hc}{\lambda} = \frac{hc.T}{b}(\text{joule})$

To convert it into eV

$E = \frac{hc}{eb}.T = \frac{1.24 \times 10^{-6}}{2.9 \times 10^{-3}} \times T$

(S) When $T = 10000K$

$E = \frac{1.24 \times 10^{-6}}{2.9 \times 10^{-3}} \times 1000 = 4.28 eV$

Energy sufficient to photo electric effect for metal having work function 4eV.

$\therefore S - 1$

(P) When T = 2000 K

$E = \frac{1.24 \times 10^{-6}}{2.9 \times 10^{-3}} \times 2000 = 0.856 eV$

P-3

(R) When T = 5000 K

E = 2.14eV

$\therefore$ Radiation at peak visible to human eye

R -2

For human visibility

$E = \frac{hc}{4000nm} \rightarrow \frac{hc}{700nm}$

$= \left(\frac{1240}{400} \text{to} \frac{1240}{700}\right) eV = (3.1eV \text{ to } 1.77) eV$

Q.T = 3000 K

E = 1.284 eV

$\therefore Q - 4$

[:Q.16] A series LCR circuit is connected to a $45\sin(\omega t)$ Volt source. The resonant angular frequency of the circuit is 10^5 rad s^{-1} and current amplitude at resonance is I_0. When the angular frequency of the source is $\omega = 8\times10^4\,rad\,s^{-1}$, the current amplitude in the circuit is $0.05I_0$. If L = 50 mH, match each entry in List-I with an appropriate value from List-II and choose the correct option.

	LIST-I		LIST-II
(P)	I_0 in mA	(1)	44.4
(Q)	The quality factor of the circuit	(2)	18
(R)	The bandwidth of the circuit in rad s^{-1}	(3)	400
(S)	The peak power dissipated at resonance in Watt	(4)	2250
		(5)	500

[:A] P → 2, Q → 3, R → 5, S → 1

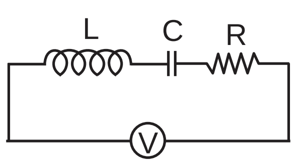
[:B] P → 3, Q → 1, R → 4, S → 2

[:C] P → 4, Q → 5, R → 3, S → 1

[:D] P → 4, Q → 2, R → 1, S → 5

[:ANS] B

[:SOLN]



$V = 45\sin(\omega t) = V_0 \sin\omega t$

$\omega_r = 10^5 \frac{rad}{s}$

$L = 50\,mH$

$I_0 = \frac{V_0}{R} = \frac{45}{R}$

When $\omega = 8\times10^4 \frac{rad}{s}$

At resonance

$\omega_r L = \frac{1}{\omega_r C}$

$\Rightarrow \quad C = \dfrac{1}{\omega_r^2 L}$

$\Rightarrow \quad C = \dfrac{1}{(10^5)^2 \times L} = \dfrac{1}{10^{10} \times 50 \times 10^{-3}}$

$C = \dfrac{1}{5 \times 10^8} = \dfrac{10}{5} \times 10^{-9} = 2\,nF$

When $\omega = 8 \times 10^4$

$X_L - X_C = \omega L - \dfrac{1}{\omega C}$

$= 8 \times 10^4 \times 50 \times 10^{-3} - \dfrac{1}{8 \times 10^4 \times 2 \times 10^{-9}}$

$= \dfrac{-9}{4} \times 10^3$

So, $X_C - X_L = \dfrac{9}{4} \times 10^3$

$\Rightarrow \quad \dfrac{45}{\sqrt{R^2 + (X_L - X_C)^2}} = 0.05 \times \dfrac{45}{R}$

$\Rightarrow \quad \boxed{R = 112.6\,\Omega}$

(P) $I_0 = \dfrac{V_0}{R} = \dfrac{45}{112.6} = 0.3996$

$I_0(mA) = 399.6\,mA$

(Q) $Q = \dfrac{\omega_r L}{R}$

$Q = \dfrac{10^5 \times 50 \times 10^{-3}}{112.6} = 44.40$

(R) $Q = \dfrac{\omega_r}{\Delta\omega}$

$\Delta\omega = \dfrac{\omega_r}{Q} = \dfrac{10^5}{44.4} = 2250$

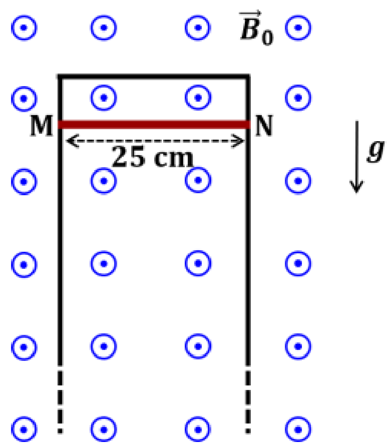
(S) $P = \dfrac{V^2}{R} = \dfrac{(45)^2}{112.5} = 18$

So, $P \to 3$; $Q \to 1$; $R \to 4$; $S \to 2$

$\therefore$ Option (B) is correct

[:Q.17] A thin conducting rod MN of mass 20 gm, length 25 cm and resistance 10 Ω is held on frictionless, long, perfectly conducting vertical rails as shown in the figure. There is a uniform magnetic field B_0 = 4 T directed perpendicular to the plane of the rod-rail arrangement. The rod is released from rest at time t = 0 and it moves down along the rails. Assume air drag is negligible. Match each quantity in List-I with an appropriate value from List-II, and choose the correct option.

[Given: The acceleration due to gravity g = 10 m s^{-2} and e^{-1} = 0.4]



LIST-I	LIST-II
(P) At t = 0.2 s, the magnitude of the induced emf in Volt	(1)0.07
(Q) At t = 0.2 s, the magnitude of the magnetic force in Newton	(2)0.14
(R) At t = 0.2 s, the power dissipated as heat in Watt	(3) 1.20
(S) The magnitude of terminal velocity of the rod in m s^{-1}	(4) 0.12
	(4) 2.00

[:A] P → 5, Q → 2, R → 3, S → 1

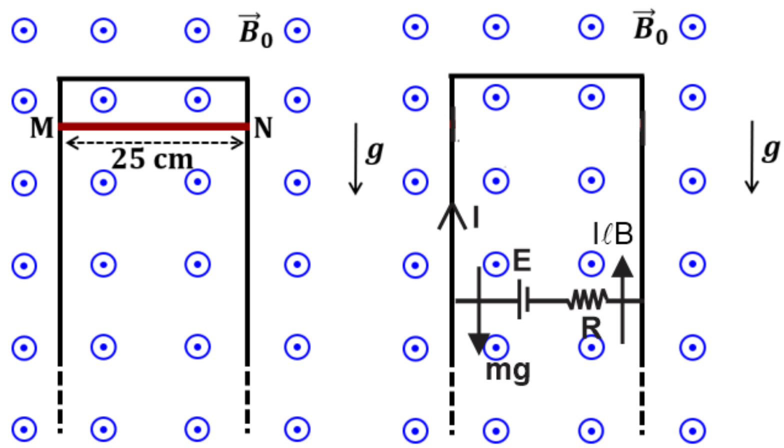
[:B] P → 3, Q → 1, R → 4, S → 5

[:C]` P → 4, Q → 3, R → 1, S → 2

[:D] P → 3, Q → 4, R → 2, S → 5

[:ANS] D

[:SOLN]



Force at any time 't'

$$F = mg - I\ell B$$

$$m\frac{dv}{dt} = mg - \left(\frac{VB\ell}{R}\right)\ell B$$

$$m\frac{dv}{dt} = mg - V\frac{B^2\ell^2}{R}$$

$$\Rightarrow \quad \frac{dv}{dt} = g - \frac{VB^2\ell^2}{Rm}$$

$$\frac{dv}{dt} = 10 - 5V$$

$$\int_0^v \frac{dv}{10-5V} = \int_0^t dt$$

$$\Rightarrow \boxed{V = 2(1 - e^{-5t})}$$

(P) $E = VB\ell$

$E = 1.2$

(Q) $F = I\ell B$

$= \frac{VB^2\ell^2}{R}$

$= 0.12$

(R) $P = I^2R$

$= \left(\frac{VB\ell}{R}\right)^2 R = \frac{V^2B^2\ell^2}{R}$

$P = 0.144$

(S) $V = 2(1 - e^{-5t})$

$t \to \infty$

V = 2 m/s

So, $P \to 3$; $Q \to 4$; $R \to 2$; $S \to 5$

$\therefore$ Option (D) is correct